7th ITER International School on
High performance computing in fusion science
Aix-Marseille University, Aix-en-Provence, France
2014-08-28

Integrated Modelling and Simulation of Toroidal Plasmas

Atsushi Fukuyama
Department of Nuclear Engineering, Kyoto University

Outline

1. Integrated modelling of toroidal plasmas
2. Data exchange in integrated simulation
3. Integrated tokamak modelling code TASK
4. Various level of transport modelling
5. Full wave analysis in toroidal plasmas
6. Summary

Integrated Simulation of Toroidal Plasmas

In order to

- predict the performance of future fusion devices
- optimize their operation scenario
- contribute to acceptable design of DEMO reactor

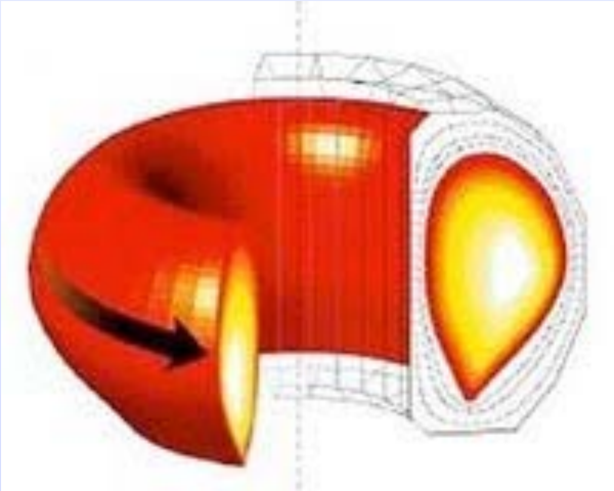
We need a reliable tool to describe

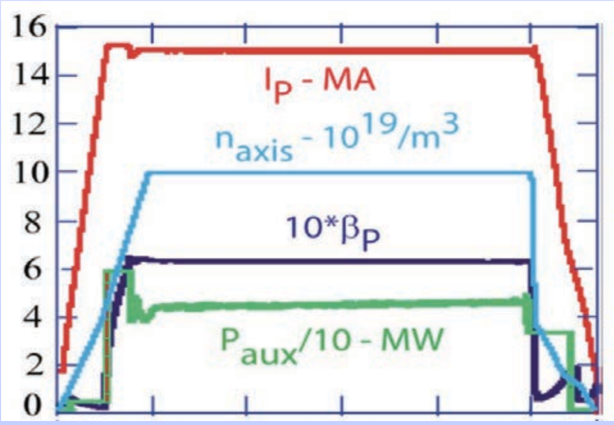
Whole plasma

- core, edge, scrape-off layer, divertor plasmas, and plasma-wall interactions

Whole discharge period

- startup, sustainment, probabilistic incidents, and shut down





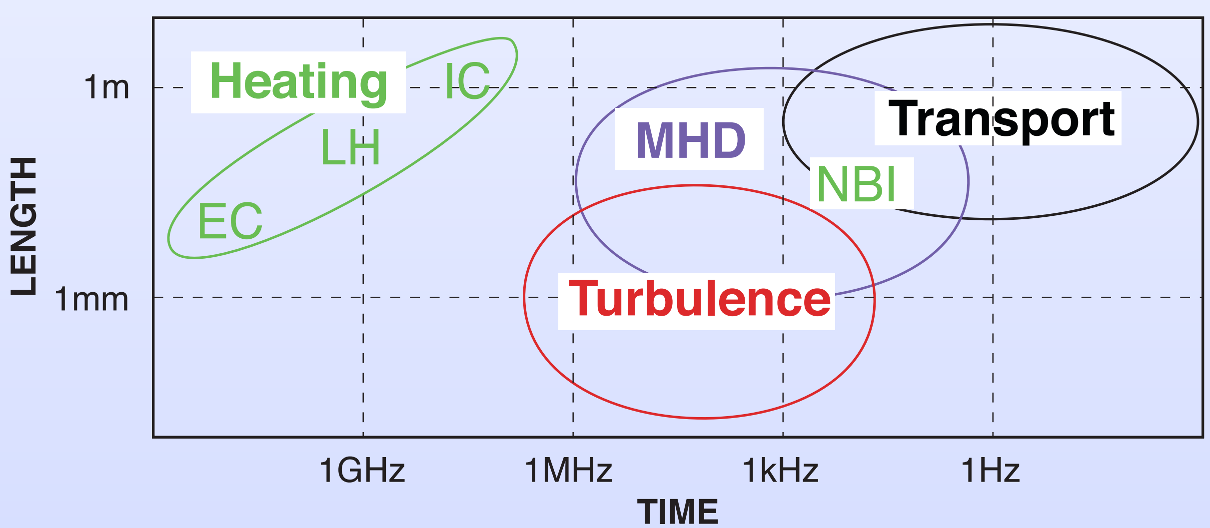
Use Case of Integrated Modelling

Device design phase	Prediction of performance Specification of components
Before experiment	Prediction of time evolution Optimization of operation scenario
During experiment	Real time analysis Between shot analysis
After experiment	Systematic analysis of experimental data Validation of physics models
Next device	Conceptual design Development of control system

Modelling of Toroidal Plasmas

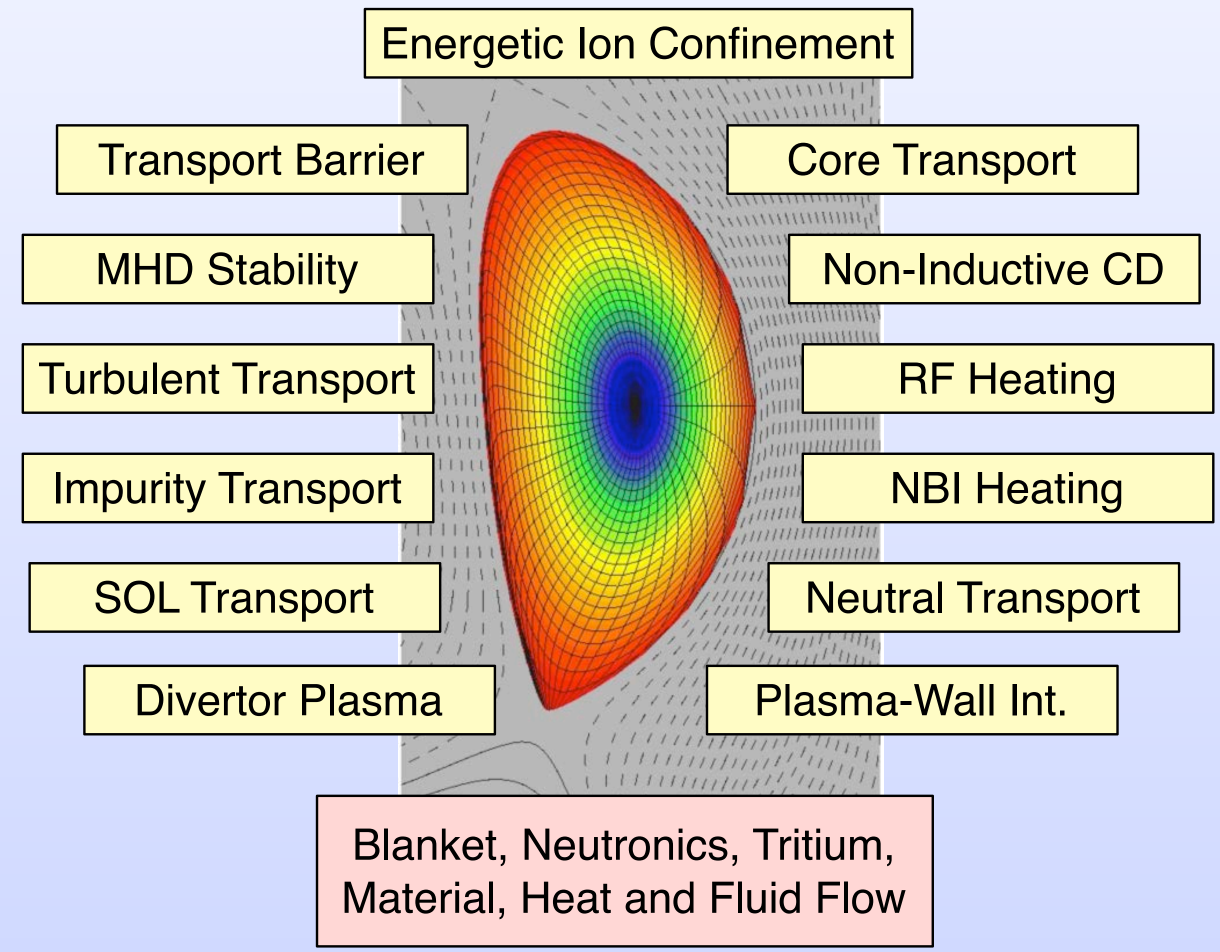
Broad range of time scale
100 GHz ～ 1000 s

Broad range of spatial scale:
10 μm ～ 10 m



One simulation code cannot cover all range.

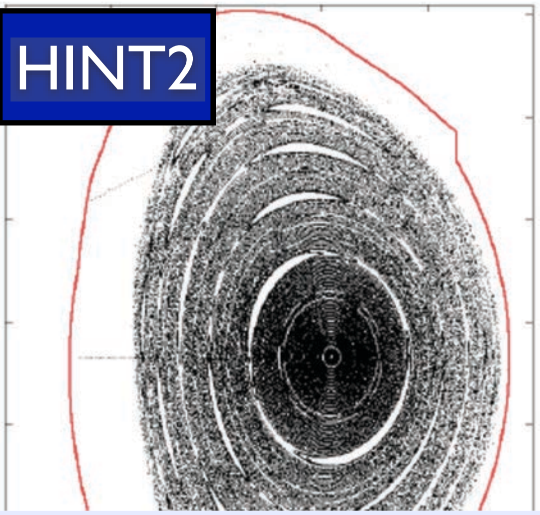
Various phenomena in toroidal plasmas

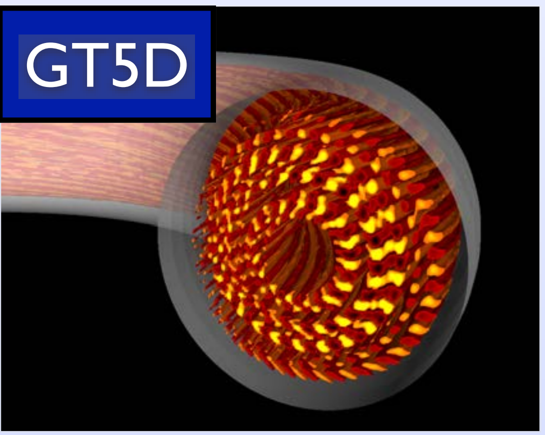


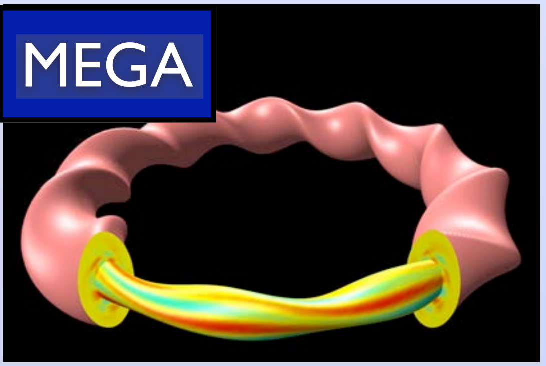
✻ Wide range of time scale, spatial scale, and understanding

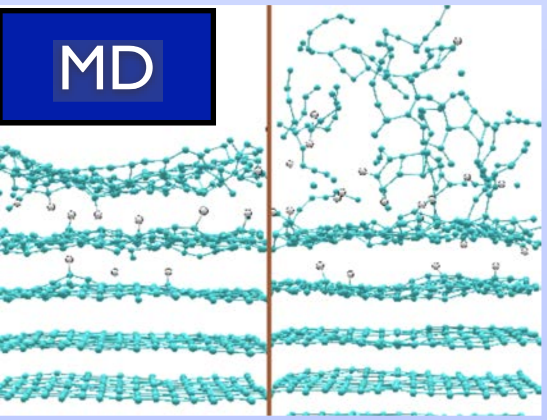
- Integrated simulation combining modelling codes
- Various levels of physics model

Structure of Integrated Modelling









Integrated Modeling
- Diagnostic model
- Control model

Equilibrium: (3D MHD Equilibrium)
- Rotation, Anisotropy
- Magnetic Island
- Fixed boundary / Free boundary / Time evolution

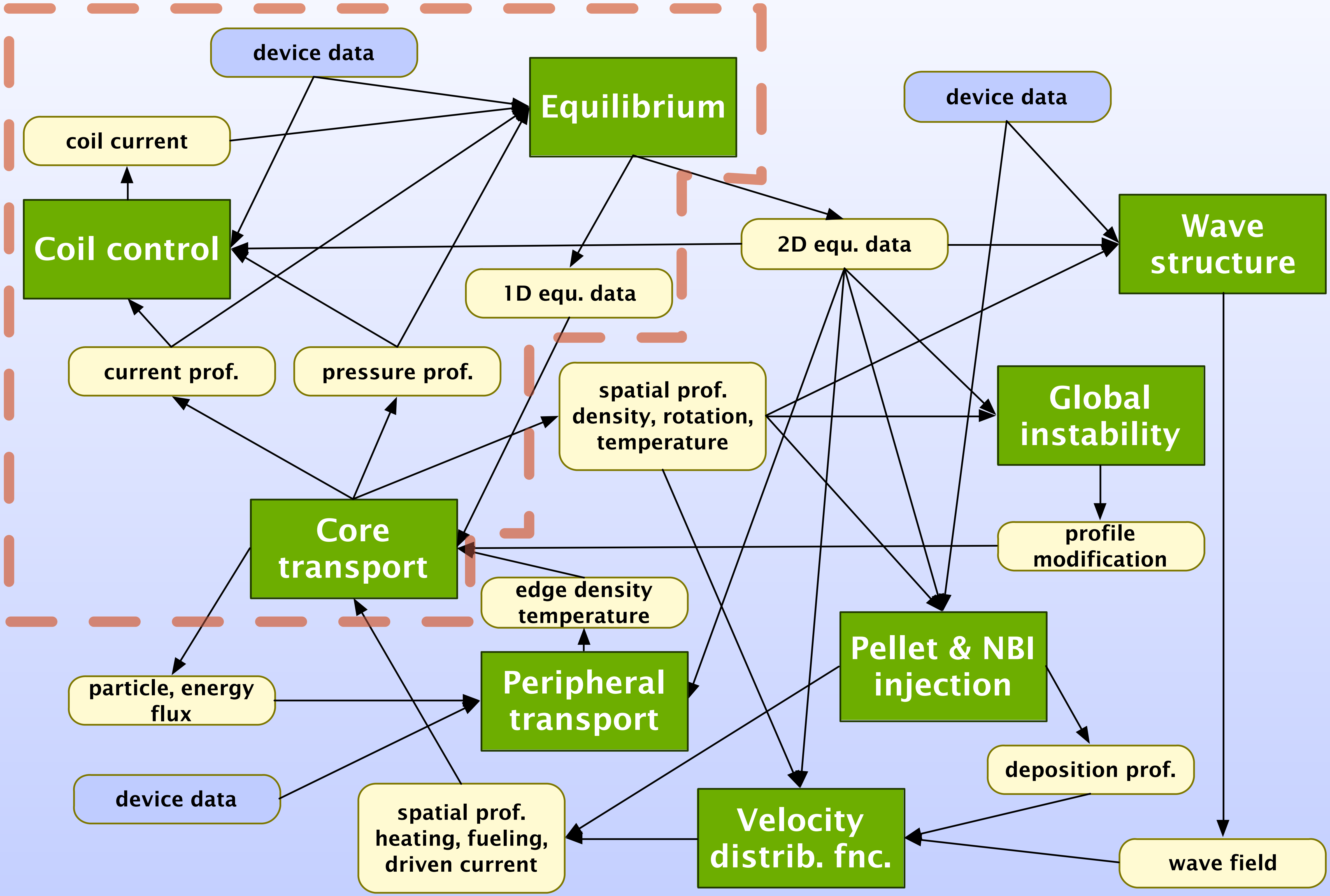
Core Plasma Transport: (Turbulence Simulation)
- Neoclassical Transport
- Neutral Transport
- Turbulent Transport
- Impurity Transport
- Radiation Transport
- Energetic Particles
- Diffusive transport / Dynamic transport / Kinetic transport / Particle orbit

Global Stability: (Nonlinear MHD)
- Linear Stability
- Plasma Deformation
- MHD stability / Kinetic stability

Source:
- NBI
- Wave: IC, LH, EC
- Pellet
- Ray/beam tracing / Full wave / Pellet ablation

Peripheral Plasma Transport: (Turbulence Simulation, Material Simulation)
- Collisional Transport
- Neutral Transport
- Turbulent Transport
- Impurity Transport
- Plasma-Wall Interaction
- Radiation Transport
- Fluid model / Kinetic model / Particle model

Structure of Toroidal Plasma Simulation



Desired features of Integrated Code

- **Modular structure**
  - **Easier maintenance of components**
    - Addition of new models, update of old models
  - **Various levels of analyses:**
    - Quick, Standard, Precise, Rigorous
- **Unified interface**
  - **Data set for information exchange**
  - **Program interface for data exchange**
  - **File interface for data storage**
  - **User interface for easier learning**
- **High usability**
  - **Portability: Various computational environment**
  - **Source accessibility: More user, easier maintenance**
  - **Visualization: Understanding of phenomena**
- **High performance**
  - **Parallel processing for large-scale and fast computation**

Integrated Modelling Activities

- **JA: BPSI**
  - **Burning Plasma Simulation Initiative**
  - **Data structure and data interface: BPSD**
  - **Execution control interface: BPSX**
- **EU: ITM TF**
  - **Integrated Tokamak Modelling - Task Force**
  - **Data model: CPO** (Consistent Physical Objects)
  - **Code interface: UAL** (Universal Access Layer)
- **ITER: IM Programme**
  - **IMAS: Integrated Modelling Analysis Suits**
  - **IM standards and guideline**
  - **ITER Data model**
    - Data exchange between modules
    - Description of device (coils, actuators, diagnostics)
    - Experimental and simulation data storage

Data exchange between components: BPSD

- **Purpose**
  - **Standard dataset: Specify set of data**
  - **Specification of data exchange interface:** initialize, set, get
  - **Specification of file i/o interface:** save, load
- **Policy of BPSD**
  - **Minimum and Sufficient Dataset**
    - To minimize the data to be exchanged
    - Mainly profile data
    - Routines to calculate global quantities
  - **Minimum Arguments in Interfaces**
    - To maximize flexibility
    - Use structured data
    - Only one dataset in the arguments of an interface
  - **Minimum Kinds of Interfaces**
    - To make modular programming easier
    - Use function overloading

BPSD Data Exchange Interface

- **Standard dataset**: Specify data to be stored and exchanged
  - **Data structure**: Derived type (Fortran95): structured type

e.g.

time	`plasmaf%time`
number of grid	`plasmaf%nrmax`
number of species	`plasmaf%nsamax`
normalized radius	`plasmaf%rho(nr)`
Species specifier	`plasmaf%ns(nsa)`
plasma density	`plasmaf%data(nr,nsa)%density`
plasma temperature	`plasmaf%data(nr,nsa)%temperature`

- **Specification of API**:
  - **Program interface**

e.g.

Set data	`bpsd_set_data(plasmaf,ierr)`
Get data	`bpsd_get_data(plasmaf,ierr)`
Save data to file	`bpsd_save(ierr)`
Load data from file	`bpsd_load(ierr)`

  - **BPSD data file** (bpsddata): Binary file of all existing bpsd data

BPSD Standard Dataset

Category	Name	EQ	TR	TX	FP	WR	WM	DP
Shot data	bpsd_shot_type	–	–	–	–	–	–	–
Device data	bpsd_device_type	in	in	in	in			
1D equilibrium data	bpsd_equ1D_type	out	in	in	in			
2D equilibrium data	bpsd_equ2D_type	out			in	in	in	in
1D metric data	bpsd_metric1D_type	out	in	in	in			
2D metric data	bpsd_metric2D_type	out			in	in	in	in
Plasma species data	bpsd_species_type	in	in	in	in			in
Fluid plasma data	bpsd_plasmaf_type	in	out	out	i/o			in
Kinetic plasma data	bpsd_plasmak_type				out			in
Transport matrix data	bpsd_trmatrix_type		i/o					
Transport source data	bpsd_trsource_type		i/o	i/o	i/o	out	out	
Dielectric tensor data	bpsd_dielectric_type					in	in	out
Full wave field data	bpsd_wavef_type				in	out		
Ray tracing field data	bpsd_waver_type				in		out	
Beam tracing field data	bpsd_waveb_type				in		out	
User defined data	bpsd_0/1/2ddata_type	–	–	–	–	–	–	–

BPSD Code Interface

- **bpsd_set_data(data,ierr):**
  - **Copy data into internal dataset**
- **bpsd_get_data(data,ierr):**
  - **Copy of interpolate data fram internal dataset**
    - If nrmax=0, copy data;
    - otherwise interpolate for given mesh.
- **bpsd_save(ierr):**
  - **Save all BPSD data into a file**
  - **Name of the file is optional.**
- **bpsd_load(ierr):**
  - **Load all BPSD data from a file**
  - **Name of the file is optional.**
- **Interface for history archiving is under consideration.**

Several Approaches on Workflow

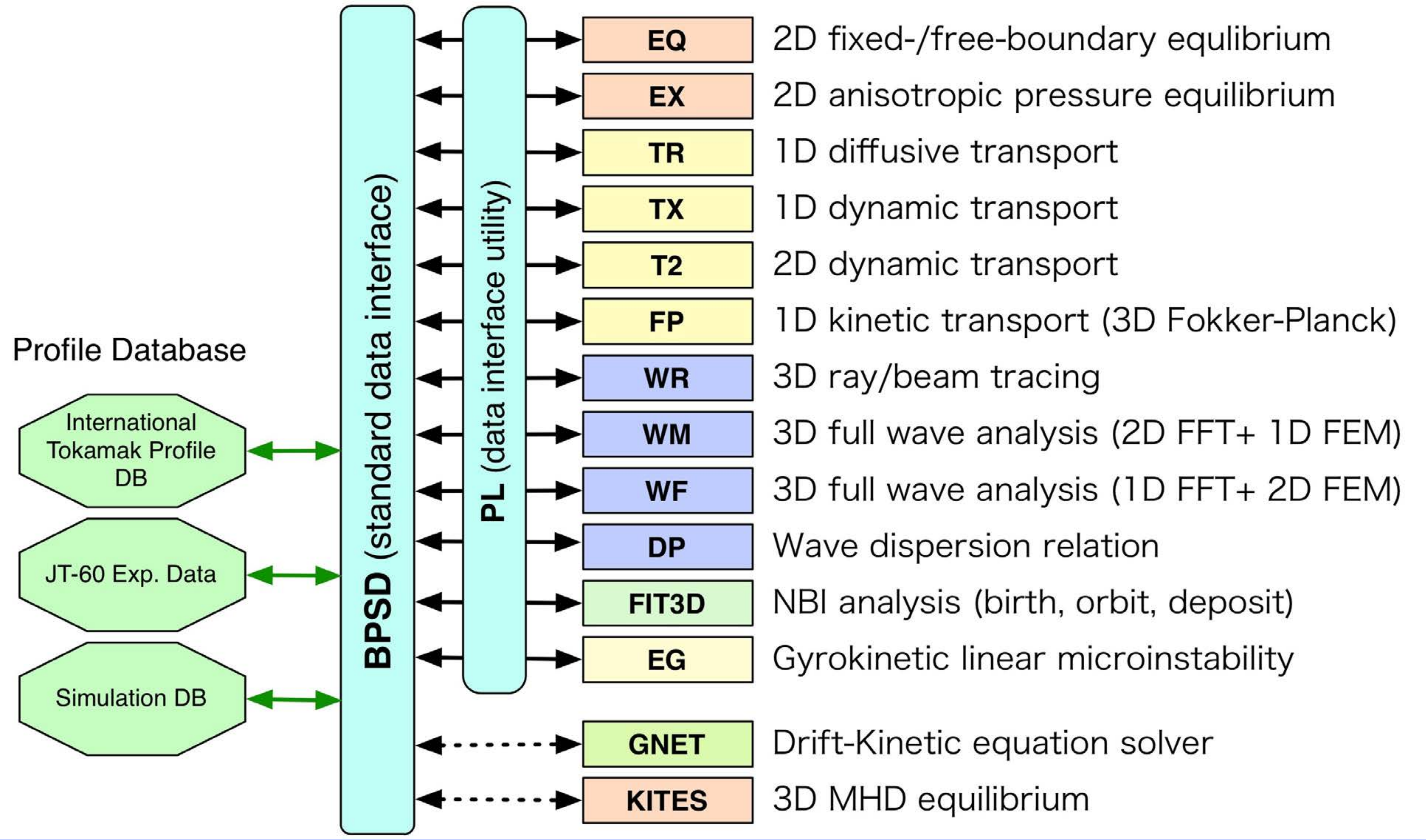
- **Monolithic code approach:** original approach
  - **Memory-based data exchange**
    - Template: call bpsd_get_data
    - calculation
    - call bpsd_set_data
- **Command approach:** for script and workflow tool
  - **File-based data exchange**
    - Template: call bpsd_load ← bpsddata
    - call bpsd_get_data
    - calculation
    - call bpsd_set_data
    - call bpsd_save → bpsddata
- **Pre- and post- process approach:** no modification of the code
  - **Data conversion**
    - Template pre-process: bpsddata → input file
    - run code
    - post-process: output file → bpsddata

Integrated Modelling Code: TASK

Transport Analyzing System for tokamaK

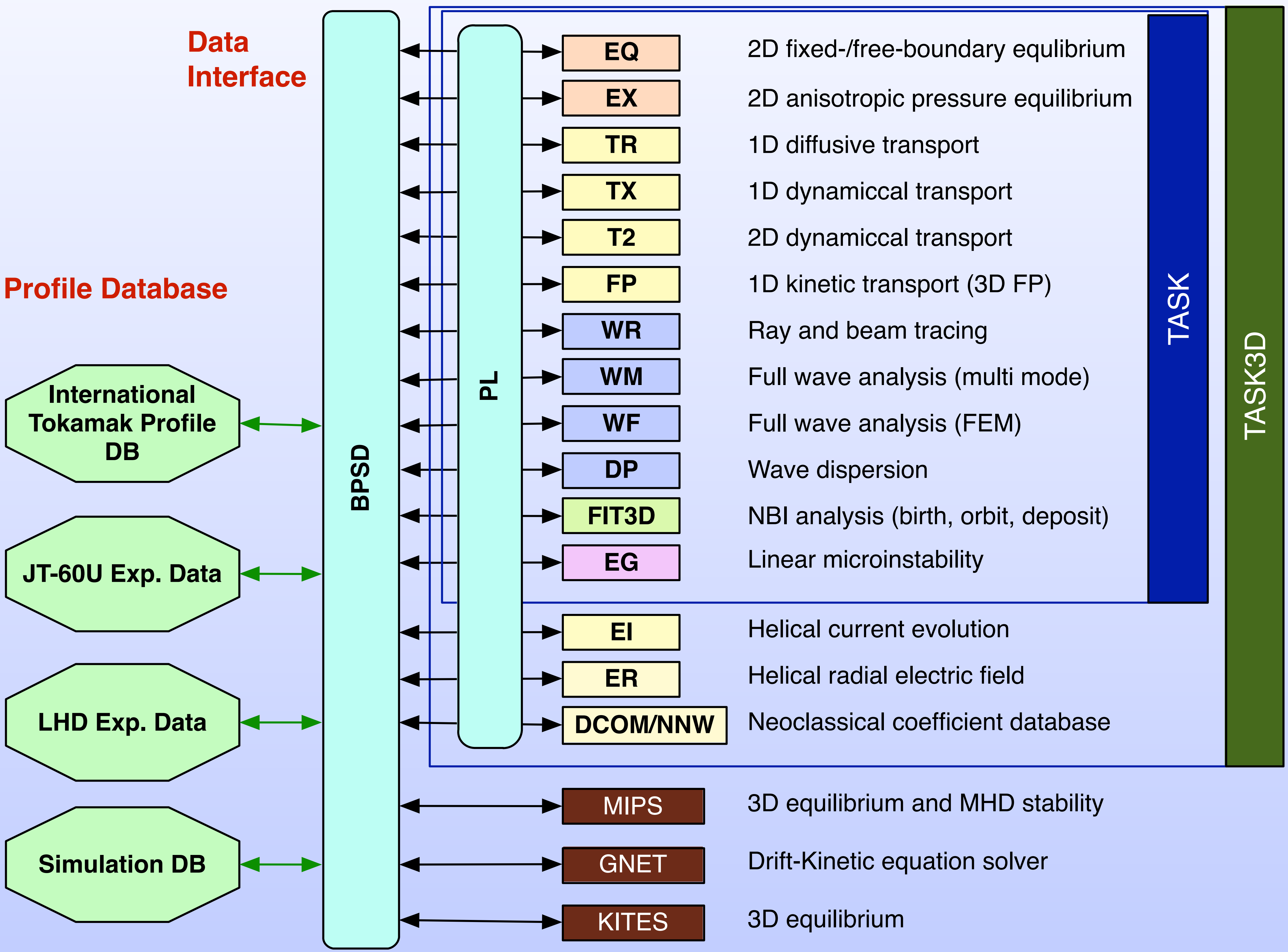
- **Core of Integrated Modelling Code in BPSI**
  - Modular structure for easier maintenance
  - Reference implementation of BPSD and BPSX
- **Various Heating and Current Drive Scheme**
  - EC, LH, IC, AW, NB
- **High Portability**
  - Most of library routines included
  - Original graphic libraries (X11, Postscript, OpenGL, SVG)
- **Development using CVS (Version control for collaboration)**
- **Open Source: http://bpsi.nucleng.kyoto-u.ac.jp/task/**
- **Parallel Processing using MPI and PETSc**

Present Structure of the TASK code and related codes



Developed since 1992, now at Kyoto University

Present Structure of TASK3D for Helical Plasmas



Various Levels of Transport Modelling

- **Fluid model**

1D Diffusive transport equation: $n(\rho,t), u_\phi(\rho,t), T(\rho,t)$	**TR**
1D Dynamic transport equation: $n(\rho,t), \boldsymbol{u}(\rho,t), T(\rho,t)$	**TX**
2D Dynamic transport equation: $n(\rho,\chi,t), \boldsymbol{u}(\rho,\chi,t), T(\rho,\chi,t)$	**T2**
3D Gyrofluid equation: $n(\rho,\chi,\zeta,t), \boldsymbol{u}(\rho,\chi,\zeta,t), T(\rho,\chi,\zeta,t)$	**BOUT**

- **Kinetic model**

Bounce-averaged drift-kinetic equation: $f(p,\theta_\mathrm{p},\rho,t)$	**FP**
Axisymmetric gyrokinetic equation: $f(p,\theta_\mathrm{p},\rho,\chi,t)$	**XGC0**
Gyrokinetic equation: $f(p,\theta_\mathrm{p},\rho,\chi,\zeta,t)$	**GT5D, GKV, GTC, GYRO**
Full kinetic equation: $f(p,\theta_\mathrm{p},\phi_\mathrm{g},\rho,\chi,\zeta,t)$	**PARASOL**

Transport Modelling in the TASK code

- **Diffusive transport equation: TASK/TR**
  - Diffusion equation for plasma density
  - Flux-Gradient relation
  - Conventional transport analysis
- **Dynamical transport equation: TASK/TX:**
  - Two-fluid equation and Maxwell's equation
  - Flux-averaged fluid equation
  - Plasma rotation and transient phenomena
- **Kinetic transport equation: TASK/FP:**
  - Drift-kinetic equation for momentum distribution function
  - Bounce-averaged Fokker-Plank equation
  - Time evolution of momentum distribution

Diffusive Transport Equation: TASK/TR

- **Transport Equation Based on Gradient-Flux Relation**:

$$\boldsymbol{\Gamma} = \overleftrightarrow{M} \cdot \partial \boldsymbol{F} / \partial \rho$$

where V: Volume, ρ: Normalized radius, $V' = \mathrm{d}V/\mathrm{d}\rho$

- **Particle transport**

$$\frac{1}{V'}\frac{\partial}{\partial t}(n_s V') = -\frac{\partial}{\partial \rho}\left(V'\langle|\nabla\rho|\rangle n_s V_s - V'\langle|\nabla\rho|^2\rangle D_s \frac{\partial n_s}{\partial \rho}\right) + S_s$$

- **Toroidal momentum transport**

$$\frac{1}{V'}\frac{\partial}{\partial t}(n_s u_{\phi s} V') = -\frac{\partial}{\partial \rho}\left(V'\langle|\nabla\rho|\rangle n_s u_{\phi s} V_{Ms} - V'\langle|\nabla\rho|^2\rangle n_z \mu_s \frac{\partial u_{\phi s}}{\partial \rho}\right) + M_s$$

- **Heat transport**

$$\frac{1}{V'^{5/3}}\frac{\partial}{\partial t}\left(\frac{3}{2} n_s T_s V'^{5/3}\right) = -\frac{1}{V'}\frac{\partial}{\partial \rho}\left(V'\langle|\nabla\rho|\rangle \frac{3}{2} n_s T_s V_{Es} - V'\langle|\nabla\rho|^2\rangle n_s \chi_s \frac{\partial T_s}{\partial \rho}\right) + P_s$$

- **Current diffusion**

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial}{\partial \rho}\left[\frac{\eta}{F R_0 \langle R^{-2}\rangle}\frac{R_0}{\mu_0}\frac{F^2}{V'}\frac{\partial}{\partial \rho}\left(\frac{V' B_\theta}{F}\left\langle \frac{|\nabla\rho|^2}{R^2}\right\rangle\right) - \frac{\eta}{F R_0 \langle R^{-2}\rangle}\langle \boldsymbol{J}\cdot\boldsymbol{B}\rangle_{\mathrm{ext}}\right]$$

Transport processes

- **Neoclassical transport**
  - **Collisional transport in a nonuniform magnetic field**
  - Radial diffusion, enhanced resistivity, bootstrap current, Ware pinch
- **Turbulent transport**
  - **Various transport models**
  - GLF23, CDBM, Bohm/gyro Bohm, TGLF, ···
- **Atomic transport**
  - **charge exchange, ionization, recombination**
- **Radiation transport**
  - **Line radiation, Bremsstrahlung, Synchrotron radiation**
- **Parallel transport**
  - **along open magnetic field lines in SOL plasmas**
- **Sources**
  - **Particle: gas puff, NBI, pellet**
  - **Momentum: NBI, waves**
  - **Heat: NBI, waves, fusion reaction**

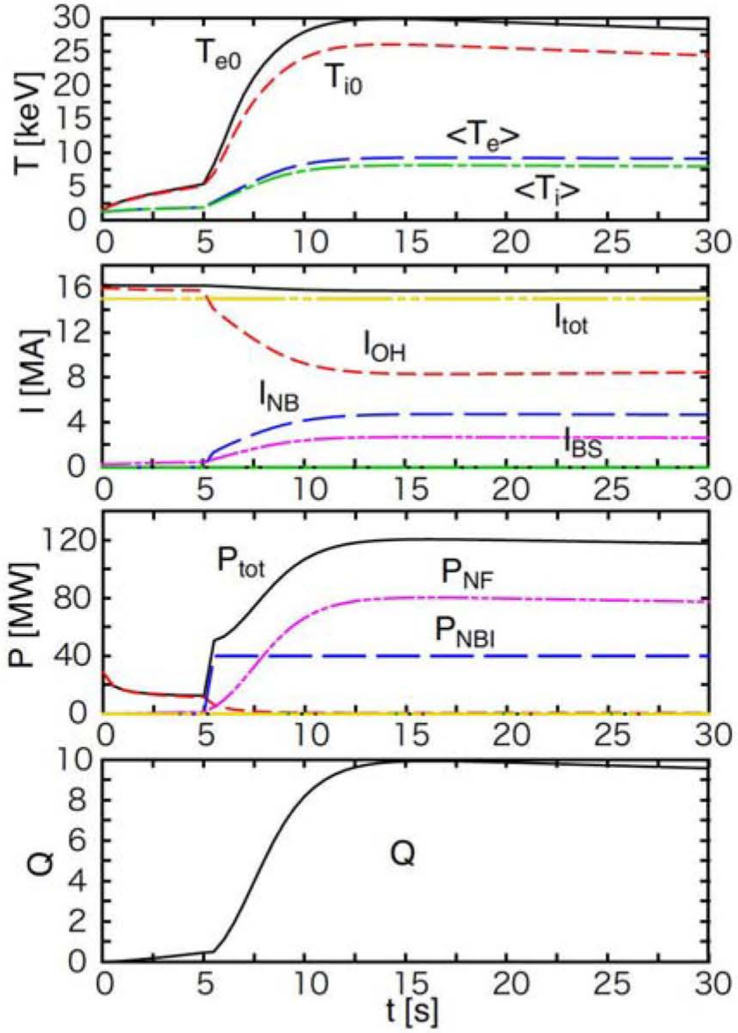
Heat Transport Simulation of ITER Scenarios

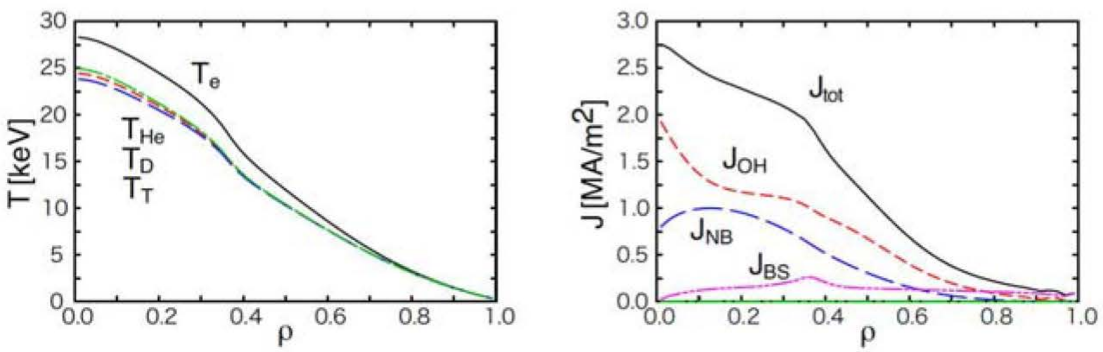
High Performance Scenario

CDBM05

$\beta_N = 1.88$

$\tau_E = 3.0$ s



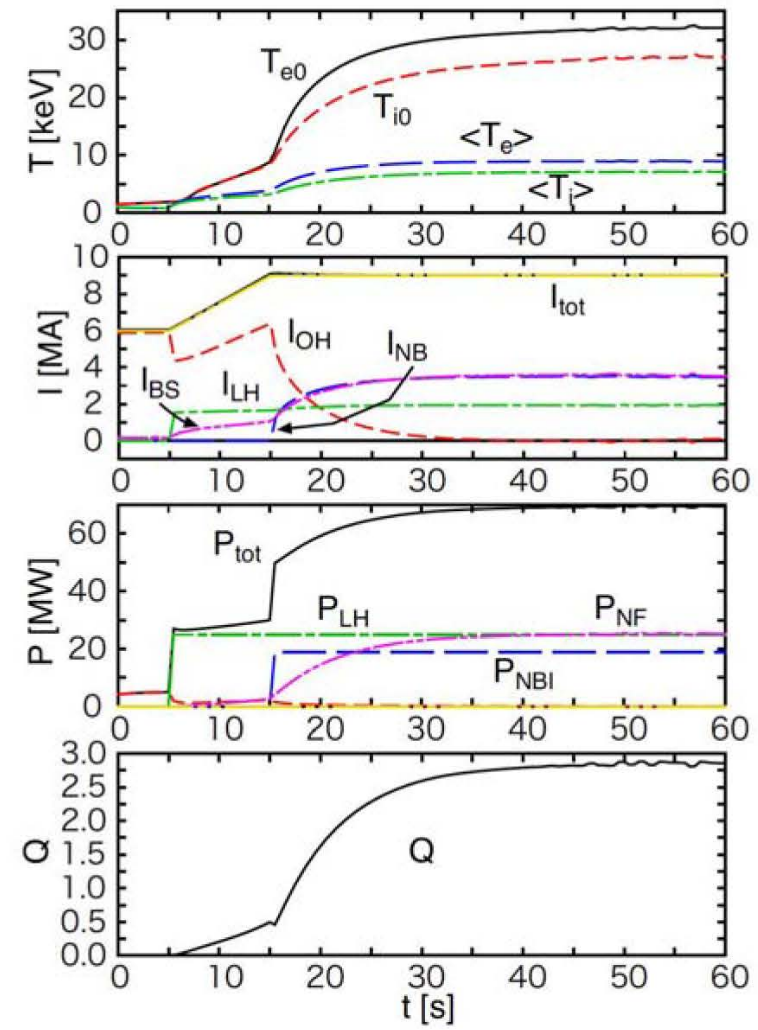


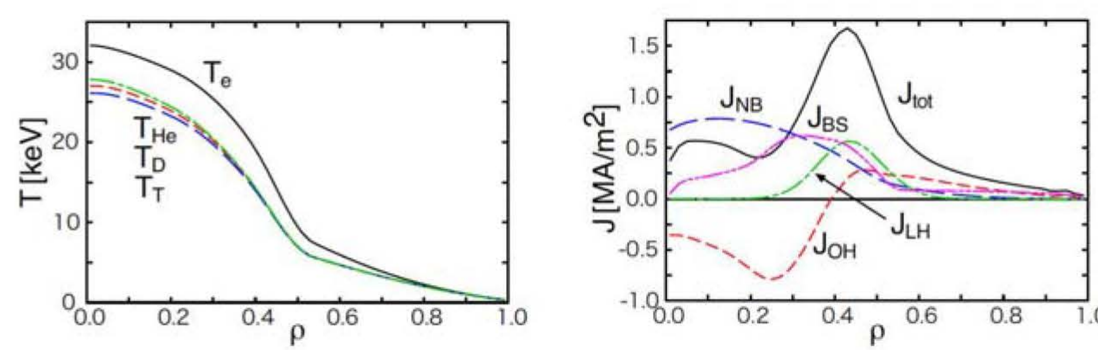
Steady State Scenario

CDBM05

$\beta_N = 1.8$

$\tau_E = 3.1$ s





1D Dynamical Transport Code: TASK/TX

- **Dynamical Transport Equations (TASK/TX)**
  - *M. Honda and A. Fukuyama, JCP 227 (2008) 2808*
  - **A set of flux-surface averaged equations**
  - **Two fluid equations for electrons and ions**
    - Continuity equations
    - Equations of motion (radial, poloidal and toroidal)
    - Heat transport equations
  - **Maxwell's equations**
  - **Slowing-down equations for beam ion component**
  - **Diffusion equations for three-group neutrals**
- **Self-consistent description of plasma rotation and electric field**
  - **Equation of motion rather than transport matrix**
- **Quasi-neutrality is not assumed.**

Dynamical Transport Equation in TASK/TX (1)

- **Continuity equations**:

$$\frac{\partial n_s}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r n_s u_{sr}) + \frac{1}{r}\frac{\partial}{\partial r}\left(r D_m v_{Ts}\frac{\partial n_s}{\partial r}\right) + S_s$$

- **Equations of motion**:

$$\frac{\partial}{\partial t}(m_s n_s u_{sr}) = -\frac{1}{r}\frac{\partial}{\partial r}(r m_s n_s u_{sr}^2) + \frac{1}{r} r m_s n_s u_{s\theta}^2 - \frac{\partial}{\partial r}(n_s T_s) + e_s n_s (E_r + u_{s\theta} B_\phi - u_{s\phi} B_\theta)$$

$$\begin{aligned}\frac{\partial}{\partial t}(m_s n_s u_{s\theta}) = &-\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 m_s n_s u_{sr} u_{s\theta}) + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^3 m_s n_s \mu_s \frac{\partial}{\partial r}\left(\frac{u_{s\theta}}{r}\right)\right] + e_s n_s (E_\theta - u_{sr} B_\phi)\\ &+ \frac{1}{r}\frac{\partial}{\partial r}\left[r D_m v_{Ts}\frac{\partial}{\partial r}(m_s n_s u_{s\theta})\right] + F_{s\theta}^{NC} + F_{s\theta}^{HNC} + F_{s\theta}^{C} + F_{s\theta}^{W} + F_{s\theta}^{L} + F_{s\theta}^{N} + F_{s\theta}^{CX}\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial t}(m_s n_s u_{s\phi}) = &-\frac{1}{r}\frac{\partial}{\partial r}(r m_s n_s u_{sr} u_{s\phi}) + \frac{1}{r}\frac{\partial}{\partial r}\left(r m_s n_s \mu_s \frac{\partial u_{s\phi}}{\partial r}\right) + e_s n_s (E_\phi + u_{sr} B_\theta)\\ &+ \frac{1}{r}\frac{\partial}{\partial r}\left[r D_m v_{Ts}\frac{\partial}{\partial r}(m_s n_s u_{s\phi})\right] + F_{s\phi}^{HNC} + F_{s\phi}^{C} + F_{s\phi}^{W} + F_{s\phi}^{L} + F_{s\phi}^{N} + F_{s\phi}^{CX}\end{aligned}$$

Dynamical Transport Equation in TASK/TX (2)

- **Heat transport equations**:

$$\frac{\partial}{\partial t}\left(\frac{3}{2}n_{\mathrm{s}}T_{\mathrm{s}}\right) = -\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{5}{2}ru_{sr}n_{\mathrm{s}}T_{\mathrm{s}} - \frac{3}{2}rn_{\mathrm{s}}\chi_{\mathrm{s}}\frac{\partial T_{\mathrm{s}}}{\partial r}\right) + e_{\mathrm{s}}n_{\mathrm{s}}(E_\theta u_{\mathrm{s}\theta} + E_\phi u_{\mathrm{s}\phi})$$

$$+ \frac{1}{r}\frac{\partial}{\partial r}\left[rD_{\mathrm{m}}v_{\mathrm{T}s}\frac{\partial}{\partial r}(n_{\mathrm{s}}t_{\mathrm{s}})\right] + P_{\mathrm{s}}^{\mathrm{C}} + P_{\mathrm{s}}^{\mathrm{L}} + P_{\mathrm{s}}^{\mathrm{R}} + P_{\mathrm{s}}^{\mathrm{RF}}$$

- **Maxwell's equation**

$$\frac{1}{R}\frac{\partial}{\partial R}(RE_r) = \frac{1}{\varepsilon_0}\sum_{\mathrm{s}} e_{\mathrm{s}}n_{\mathrm{s}}$$

$$\frac{1}{c^2}\frac{\partial E_\theta}{\partial t} = -\frac{\partial B_\phi}{\partial r} - \mu_0\sum_{\mathrm{s}} e_{\mathrm{s}}n_{\mathrm{s}}u_{\mathrm{s}\theta}$$

$$\frac{1}{c^2}\frac{\partial E_\phi}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) - \mu_0\sum_{\mathrm{s}} e_{\mathrm{s}}n_{\mathrm{s}}u_{s\phi}$$

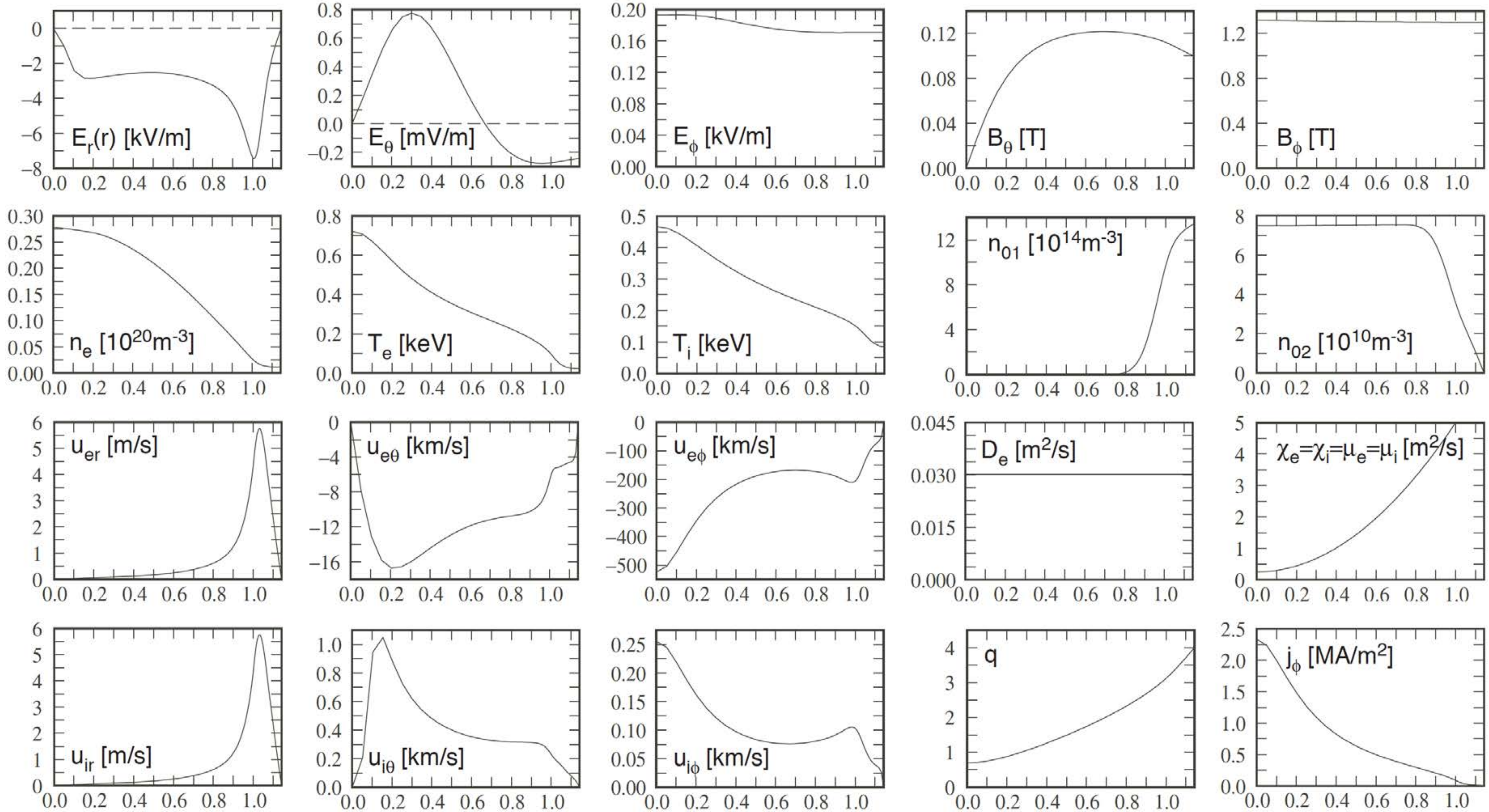
$$\frac{\partial B_\theta}{\partial t} = \frac{\partial E_\phi}{\partial r}, \qquad \frac{\partial B_\phi}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta)$$

Typical Ohmic Plasma Profiles at t = 50 ms

JFT-2M like plasma composed of electron and hydrogen

$R = 1.3\,\mathrm{m},\ a = 0.35\,\mathrm{m},\ b = 0.4\,\mathrm{m},\ B_{\phi b} = 1.3\,\mathrm{T},\ I_p = 0.2\,\mathrm{MA},\ S_{\mathrm{puff}} = 5.0 \times 10^{18}\,\mathrm{m^{-2}s^{-1}}$

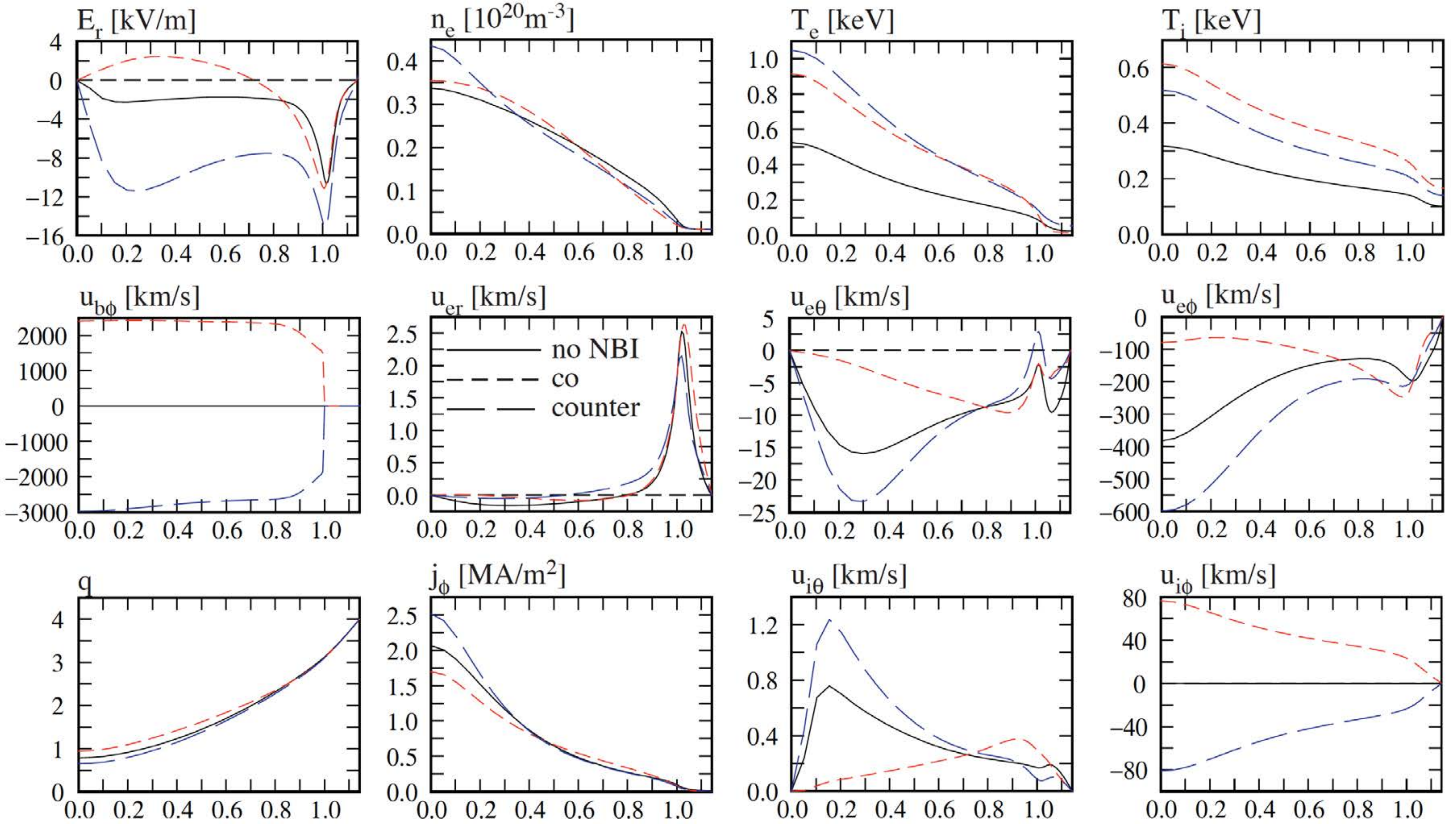
$\gamma = 0.8,\ Z_{\mathrm{eff}} = 2.0$, Fixed turbulent coefficient profile



Density Profile Modification Due to NBI Injection

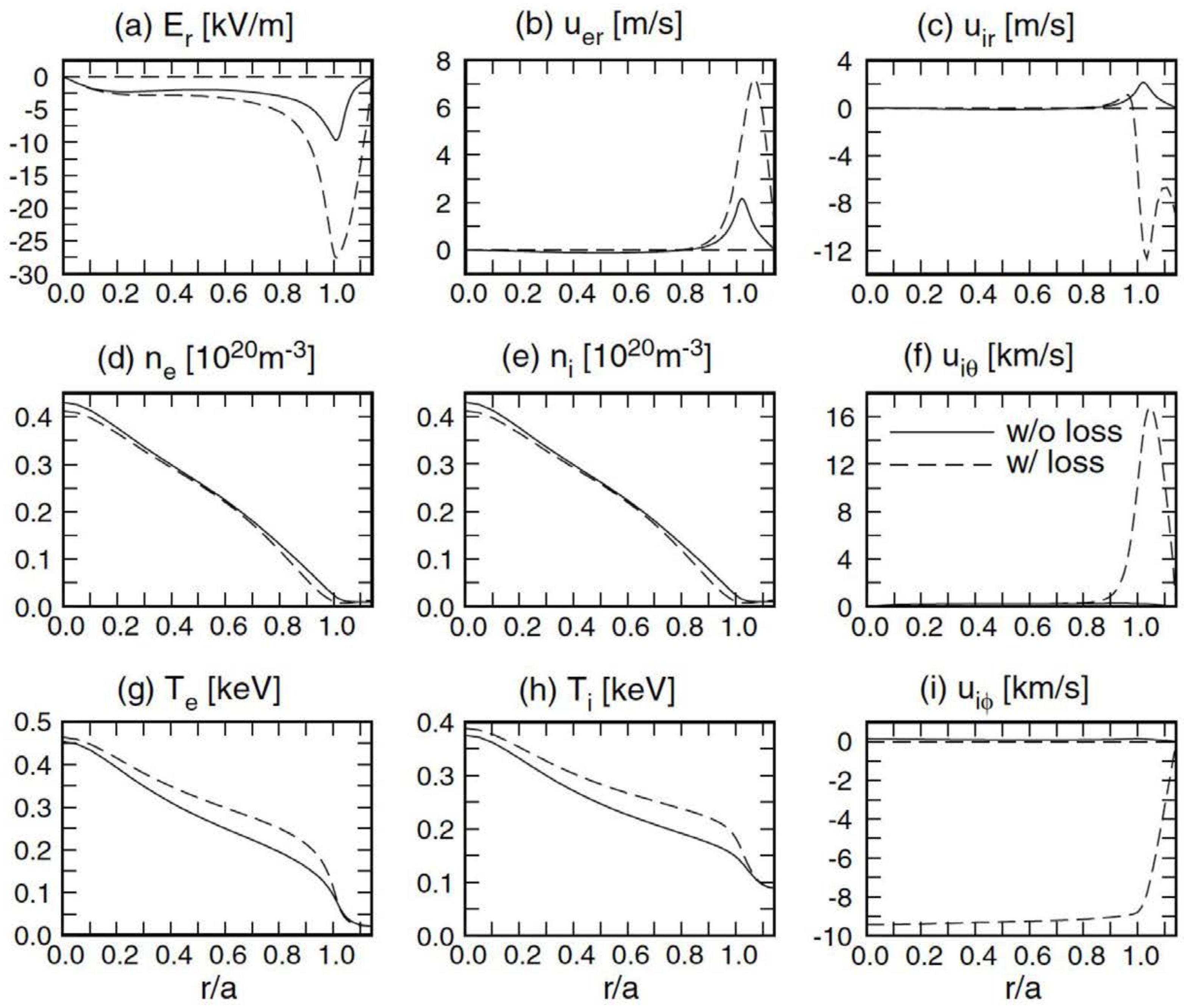
Modification of n and E_r profile depends on the direction of NBI.

Co/Counter with I_p: Density flattening/peaking



Toroidal Rotation Due to Ion Orbit Loss

- **Ion orbit loss near the edge region drives toroidal rotation**



Ref. M. Honda et al., NF (2008) 085003

Kinetic Integrated Modelling: Motivation

- **Better understanding of burning plasmas**
  - **Behavior of energetic particles**
    - generation, transport excitation
- **Analysis of momentum distribution function**
  - **Consistent analysis of heating and current drive**
    - both bulk and energetic components
    - all heating schemes
  - **Influence of energetic particles on heating processes**
    - propagation and absorption of waves
    - fusion reaction rate
  - **Modification of momentum distribution due to radial transport**
- **Modelling based on momentum distribution function is required.**

Fokker-Planck Analysis in TASK/FP

- **Multi-species momentum distribution functions**:

$$f_s(p_{||}, p_\perp, \rho, t)$$

- **Fokker-Planck equation**

$$\frac{\partial f_s}{\partial t} = E(f_s) + C(f_s) + Q(f_s) + D(f_s) + S_s$$

  - $E(f)$: **Acceleration due to DC electric field**
  - $C(f)$: **Relativistic Non-Maxwellian Coulomb collision**
  - $Q(f)$: **Quasi-linear diffusion due to wave-particle resonance**
    - Full wave analysis (TASK/WM)
    - Ray/beam tracing (TASK/WR)
    - Fixed wave field profile; Fixed diffusion coefficient profile
  - $D(f)$: **Spatial diffusion**
  - S: **Particle Source and Sink** (NBI, Fusion reaction)

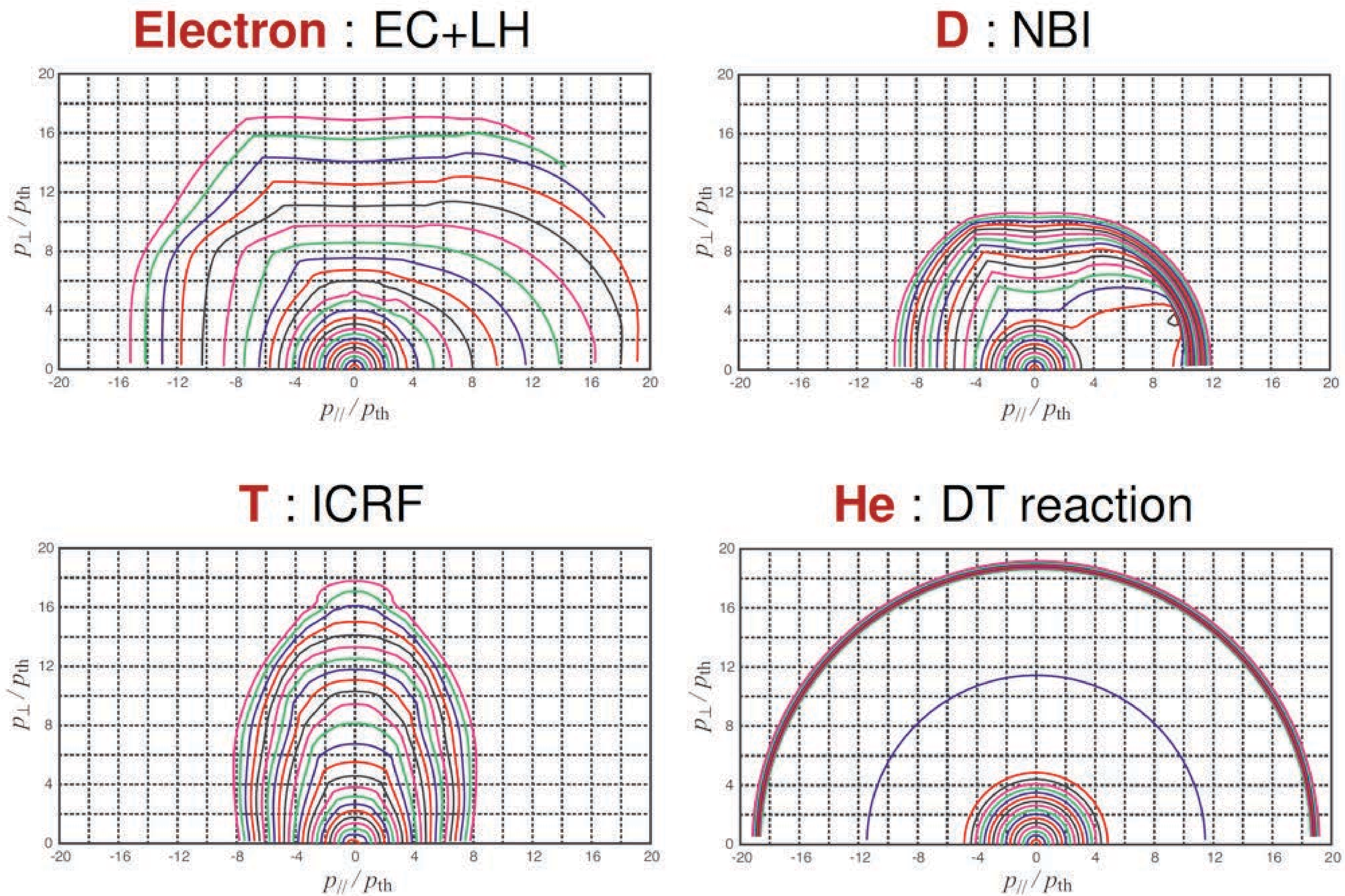
Kinetic Transport Modelling: TASK/FP

Fokker-Planck analysis of distribution function

Multi species	conservation between species
Three dimensional	2D in momentum, 1D in radial
Bounce averaged	trapped particle effect
Nonlinear collision	momentum and energy conservation
Relativistic	weakly relativistic collision term
Fusion reaction	velocity integral
Parallel processing	using parallel matrix solver PETSc library
Finite orbit size	under development
Induced EM fields	under development

Multi-Species Fokker-Planck Analysis

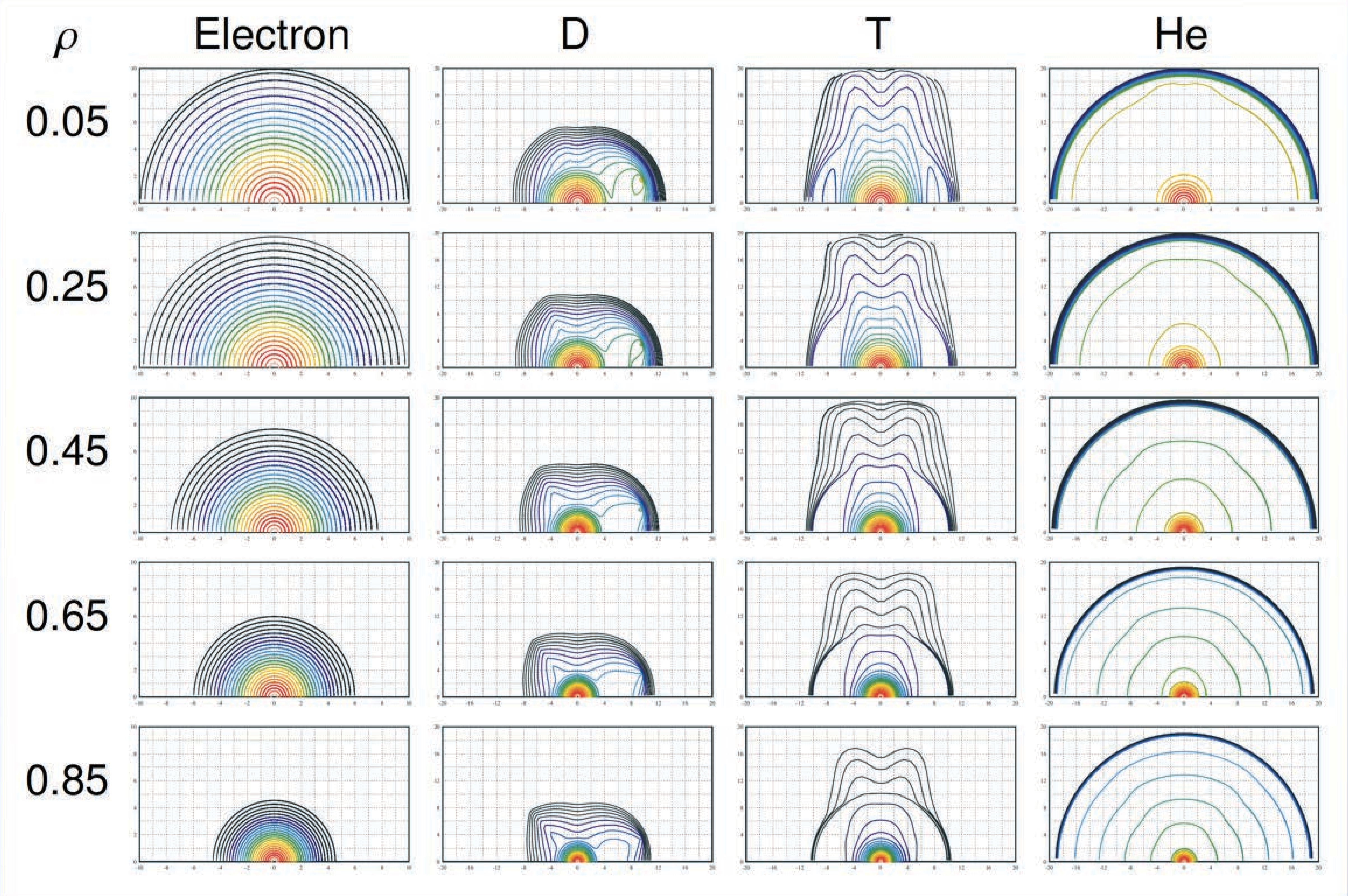
Momentum distribution functions: $f_s(p_{||}, p_\perp, \rho, t)$



Analysis of Multi-Scheme Heating in ITER Plasma

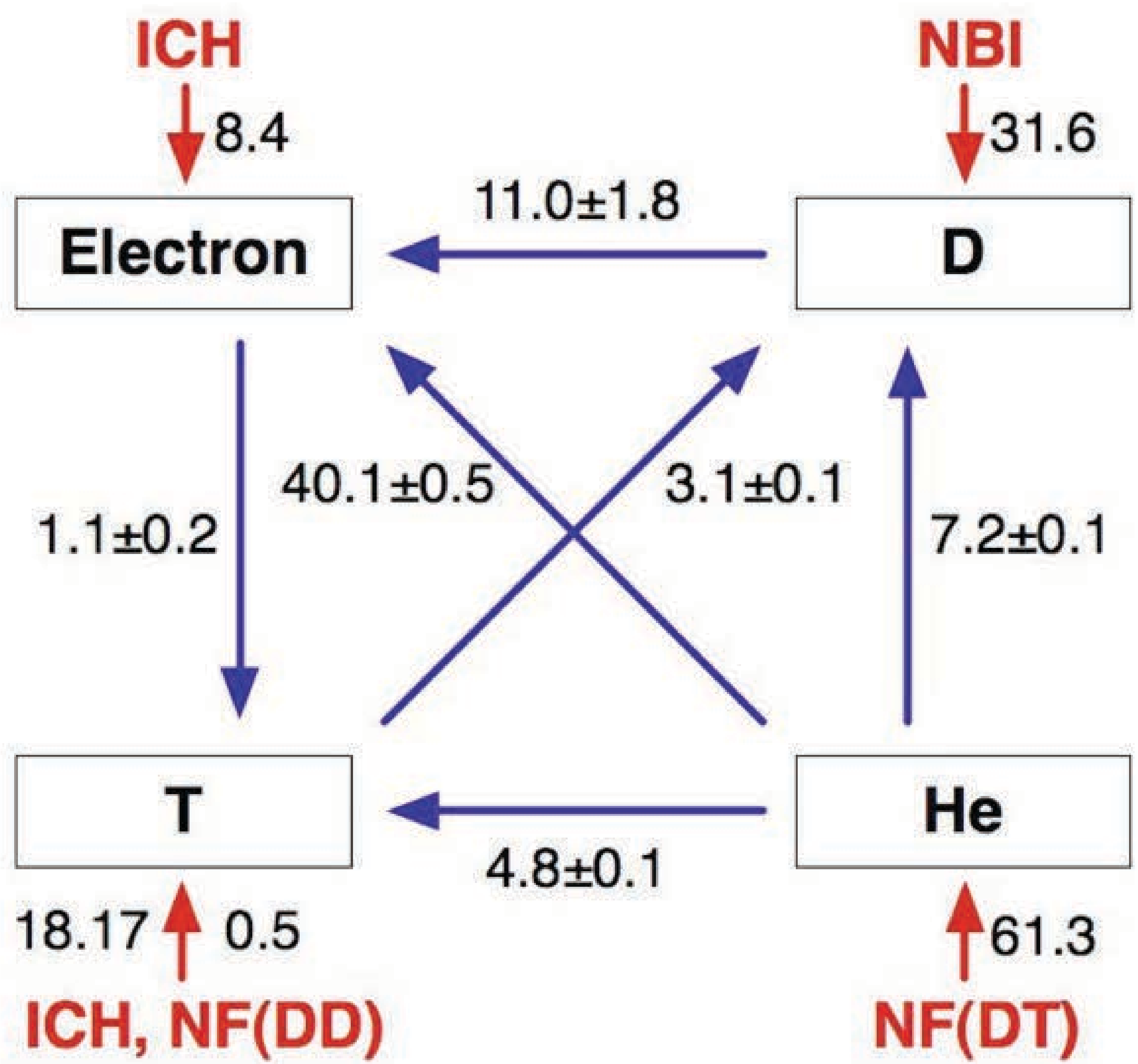
- **2D MHD equilibrium**
  - $R = 6.2\,\mathrm{m}$, $a = 2.0\,\mathrm{m}$, $\kappa = 1.7$, $\delta = 0.33$, $B_0 = 5.3\,\mathrm{T}$, $I_\mathrm{p} = 3\,\mathrm{MA}$
- **Multi species**:
  - Electron, D, T, He
- **Multi scheme heating**:
  - ICH, NBI, NF (DT, DD, TT)
- **Initial density**:
  - $n_\mathrm{e}(0) = 10^{20}\,\mathrm{m}^{-3}$, $n_\mathrm{D}(0) = 5 \times 10^{19}\,\mathrm{m}^{-3}$, $n_\mathrm{T}(0) = 5 \times 10^{19}\,\mathrm{m}^{-3}$
- **Initial temperature**:
  - $T_\mathrm{e}(0) = T_\mathrm{D}(0) = T_\mathrm{T}(0) = 20\,\mathrm{keV}$
- **Radial diffusion coefficient**: simplest model
  - $D_{rr} = 0.1(1 + 9\rho^2)\,\mathrm{m/s}$

Momentum Distribution Functions (t = 1 s)



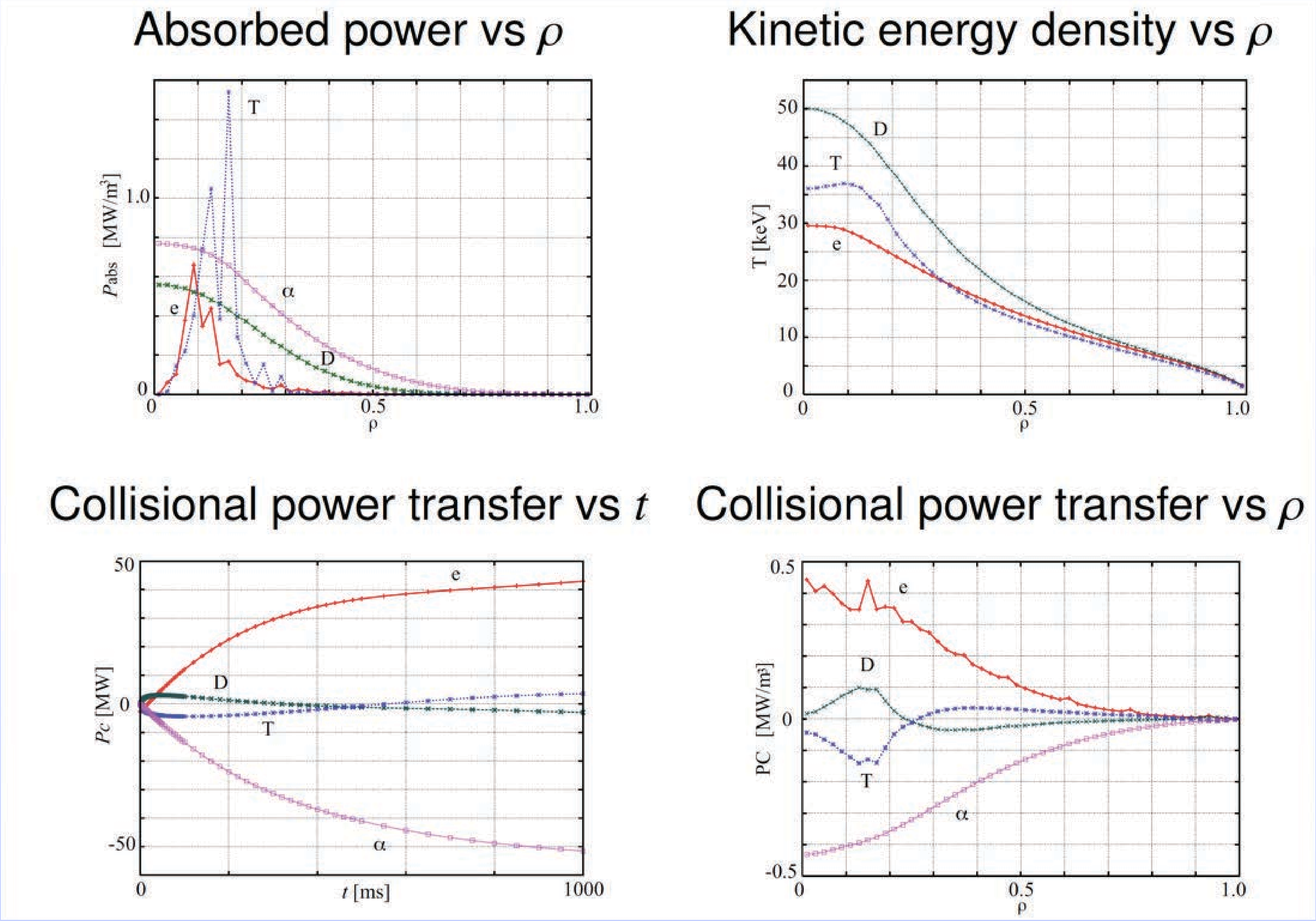
Power Transfer between Species

- Collisional power transfer



- Requires more momentum meshes for better accuracy
  - At present, typically $100 \times 100 \times 50$

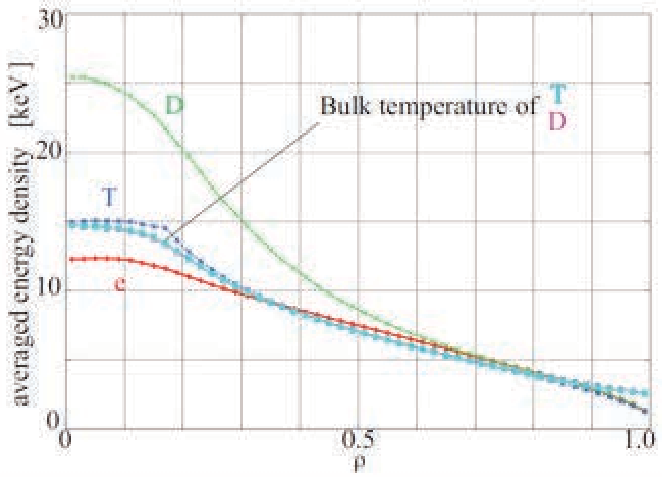
Simulation with Radial Transport

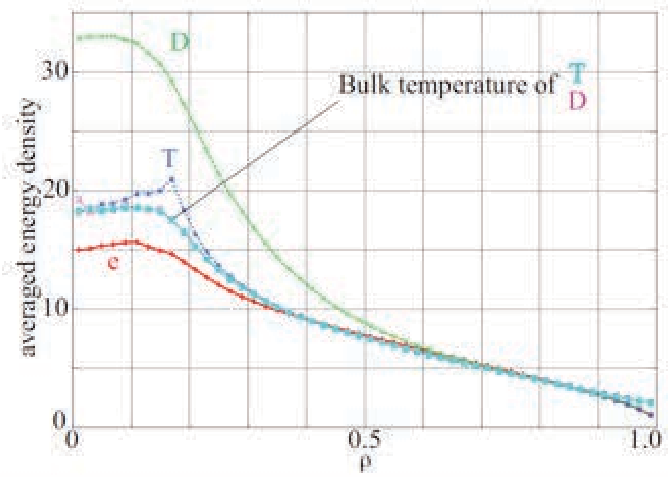


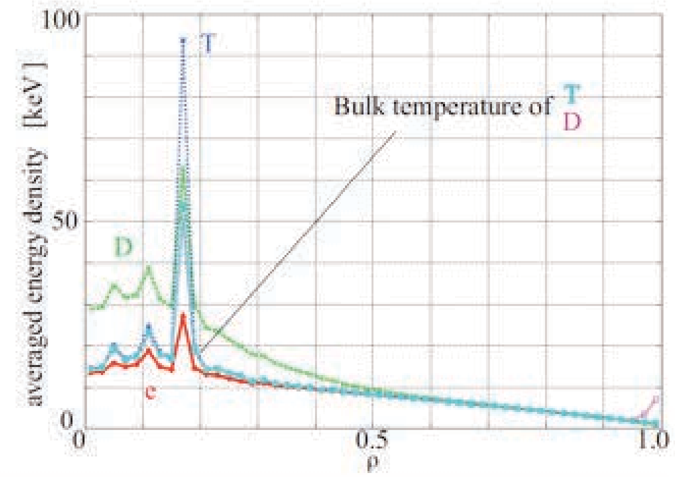
Dependence on Radial Diffusion model

Radial profile of average kinetic energy: $p' = \sqrt{p^2 + p_{\rm th0}^2}$

	p-indep. diffusion	p-dep. diffusion	no diffusion
	$D_{rr} \propto p'^0$	$D_{rr} \propto p'^{-1}$	$D_{rr} = 0$







	p-indep. diffusion	p-dep. diffusion	no diffusion
$E_{\rm KD}$ [keV]	9.57	10.82	11.72
$E_{\rm KT}$ [keV]	7.18	8.15	9.44
$E_{\rm K\alpha}$ [keV]	471.70	558.28	622.75
$P_{\rm NB}$ [MW]	31.68	31.68	31.69
$P_{\rm ICT}$ [MW]	8.95	10.87	15.28
P_{α} [MW]	23.36	32.70	36.88

- **Radial diffusion proportional to $E^{-1/2}$ reduces the alpha heating about 10 %.**

Full Wave Analysis

- **Boundary-value problem of Maxwell's equation with fixed ω**
  - $\boldsymbol{E}$: wave electric field
  - $\overleftrightarrow{\epsilon}$: dielectric tensor

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega \mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

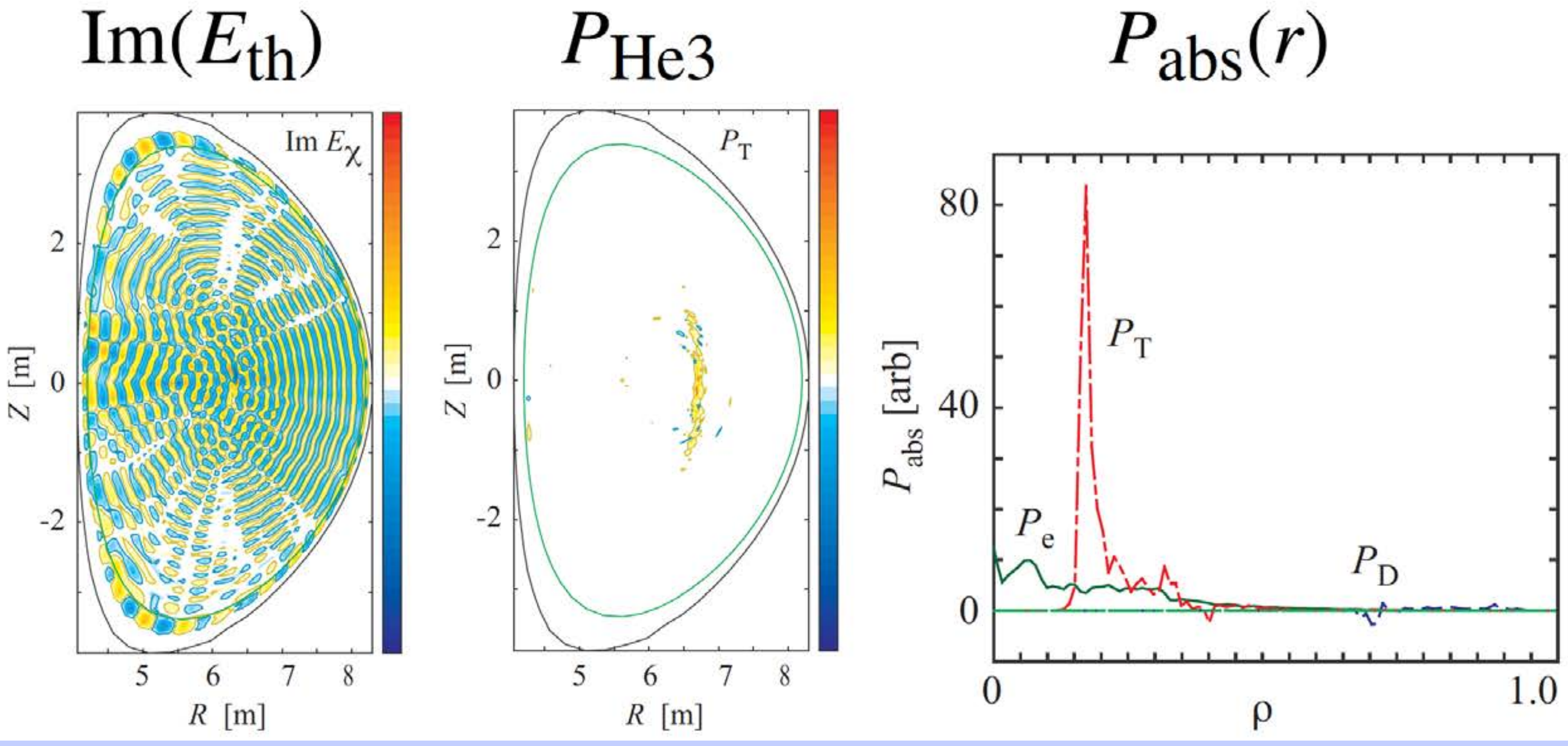
- **Merit of full wave analysis**
  - Wave length longer than the scale length of medium
  - Propagation over an evanescent layer
  - Coupling to antenna
  - Formation of standing wave
- **Method of full wave analysis**
  - **Fourier analysis**: algebraic equation
  - **Discrete differential equation**: finite difference/element method
  - **Mixture of above two methods**

Full wave analysis: TASK/WM

- **Maxwell's equation solver** as a boundary-value problem

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- **Kinetic dielectric tensor**: $\overleftrightarrow{\epsilon}$ for arbitrary $f(v)$
- **Numerical scheme**: Fourier expansion in θ and ϕ
- **Antenna excitation and eigenmode analysis**: Complex ω
- **ICRF minority heating**



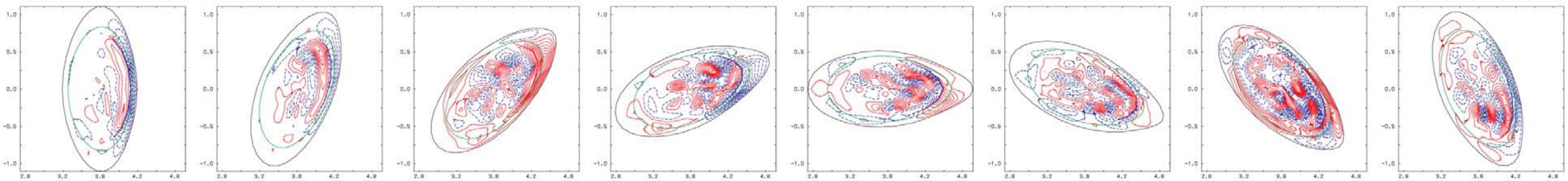
ICRF Waves in a Helical Plasma

LHD ($B_0 = 3\,\mathrm{T}$, $R_0 = 3.8\,\mathrm{m}$)

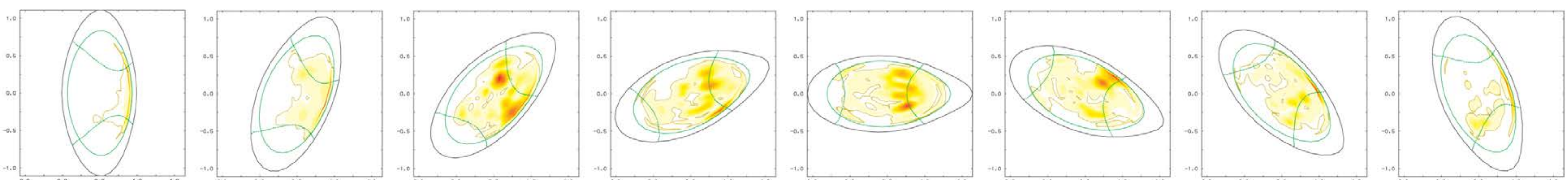
$f = 42\,\mathrm{MHz}$, $n_{\phi 0} = 20$, $n_{e0} = 3 \times 10^{19}\,\mathrm{m}^{-3}$, $n_{\mathrm{H}}/(n_{\mathrm{He}} + n_{\mathrm{H}}) = 0.235$,

$N_{\mathrm{rmax}} = 100$, $N_{\theta\mathrm{max}} = 16$ ($m = -7 \ldots 7$), $N_{\phi\mathrm{max}} = 4$ ($n = 10, 20, 30$)

Wave electric field (imaginary part of poloidal component)



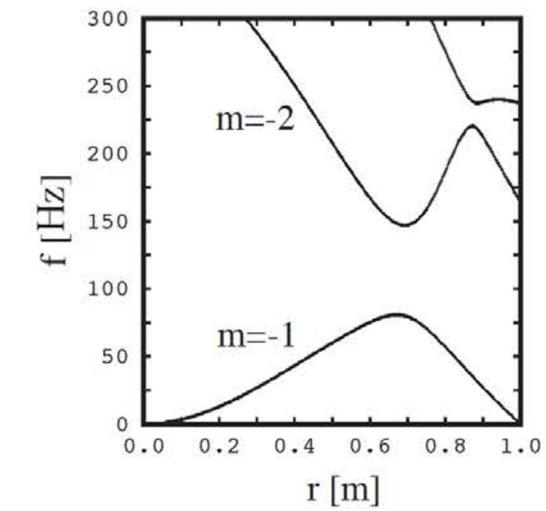
Power deposition profile (minority ion)



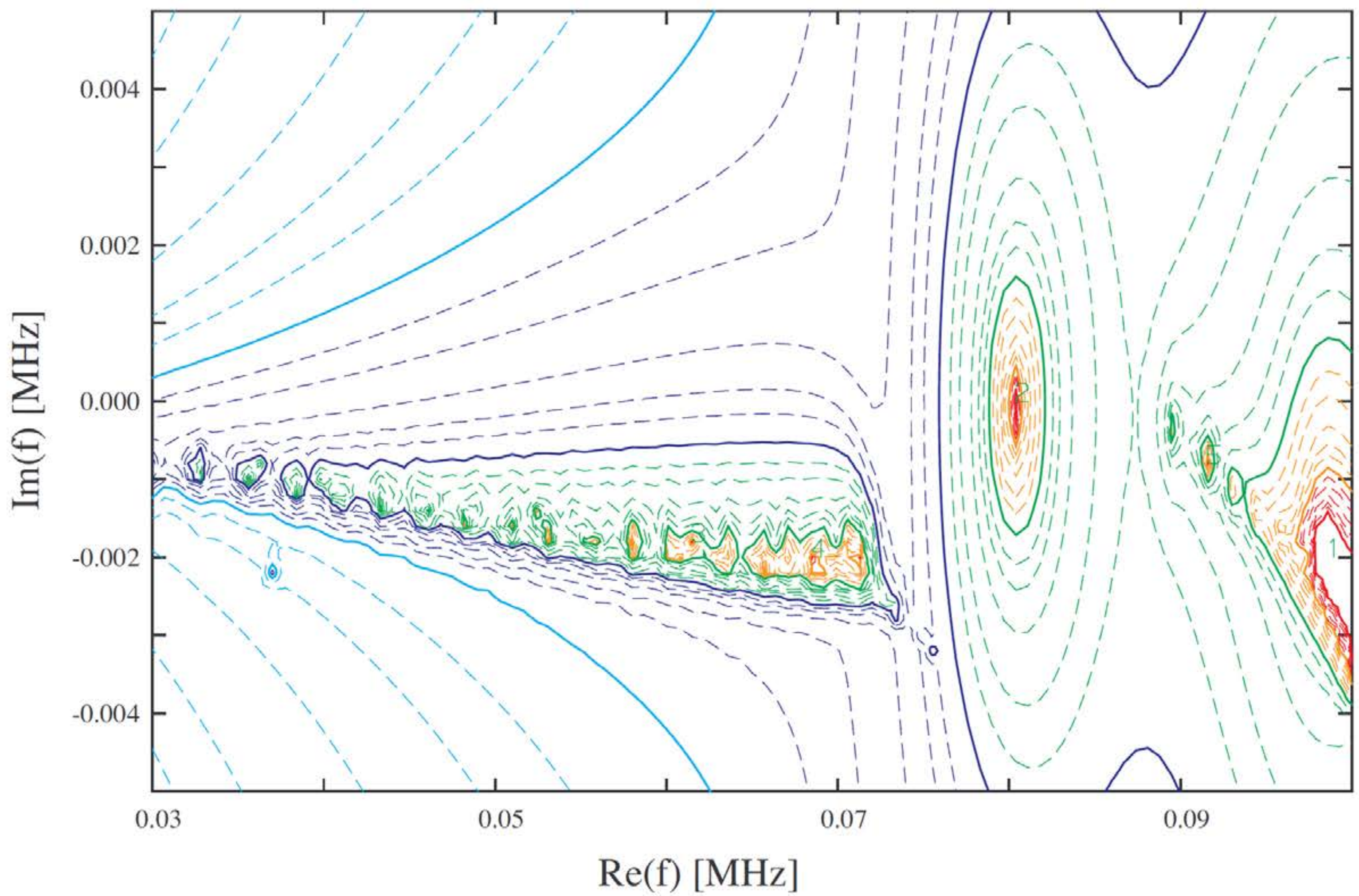
TAE Analysis with TASK/WM

- **Configuration**
  - $q(\rho) = q_0 + (q_a - q_0)\rho^2$, $q_0 = 1$, $q_a = 2$
  - Flat Density Profile

Alfvén Frequency

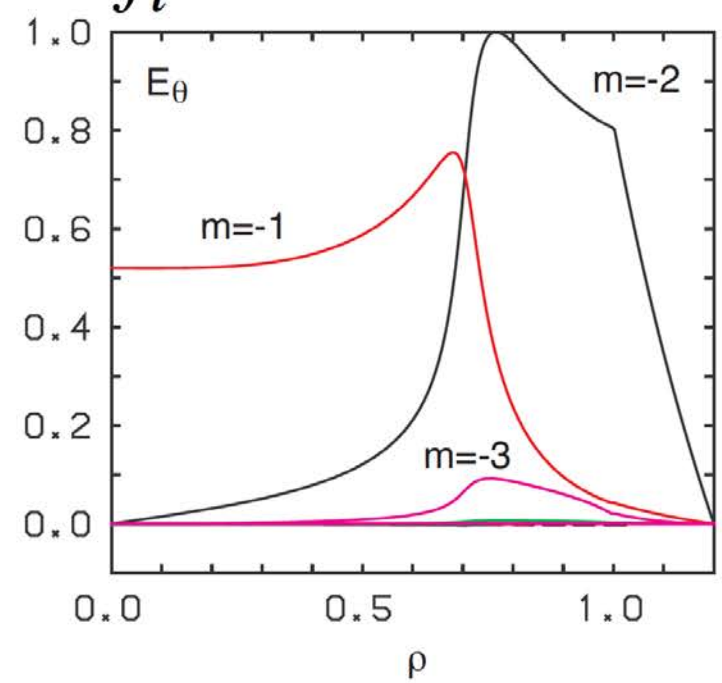


Contour of $|E|^2$ in Complex Frequency Space



Eigen function

$$f_r = 81.95\,\text{kHz}$$
$$f_i = -20.32\,\text{Hz}$$



RSAE Excitation by Energetic Particles

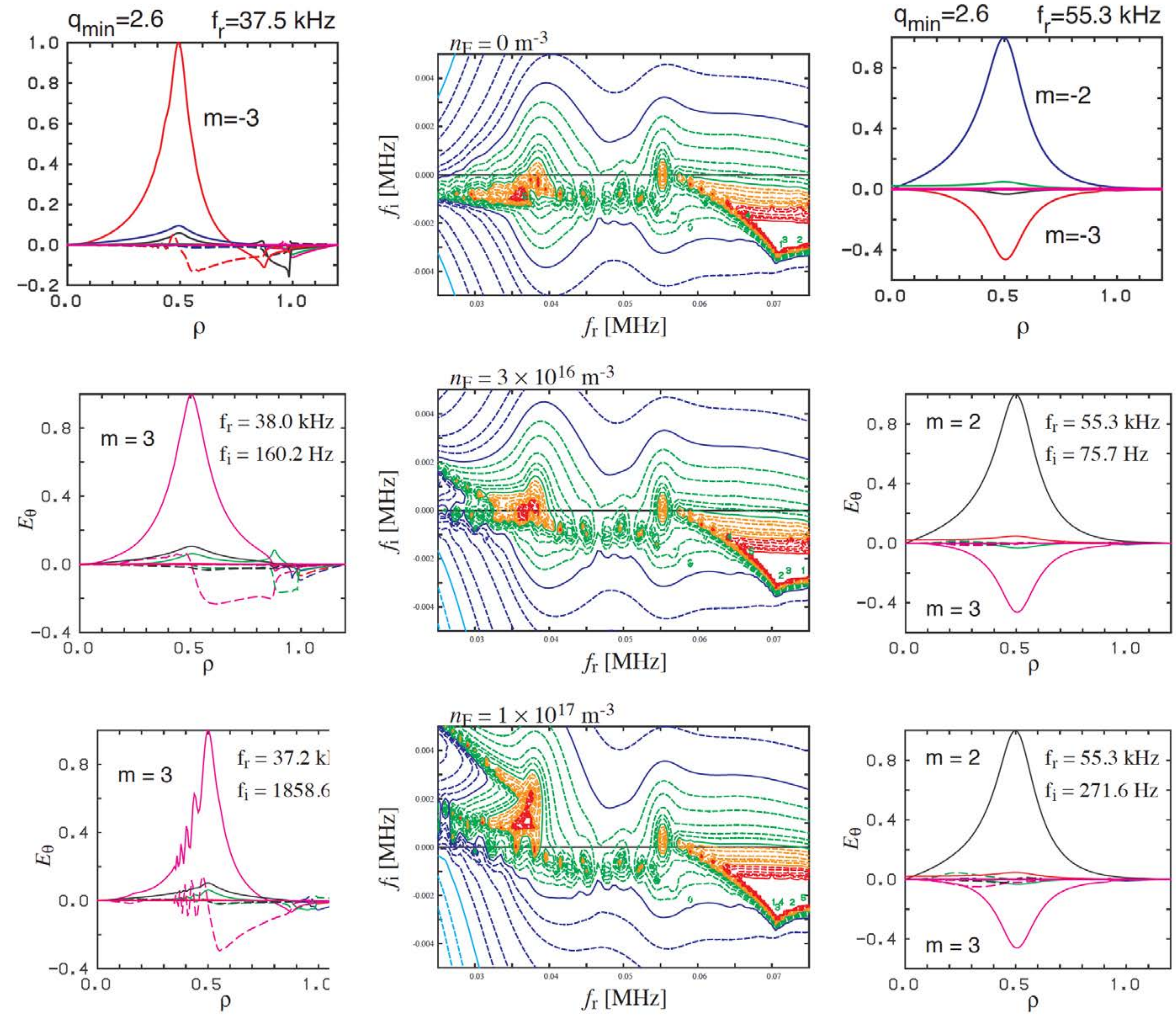
- Without EP

- With EP

  $3 \times 10^{16}\,\mathrm{m}^{-3}$
  360 keV
  0.5 m

- With EP

  $1 \times 10^{17}\,\mathrm{m}^{-3}$
  360 keV
  0.5 m



Progress in Full Wave Analysis

- **Variety of numerical schemes**

module	system	scheme
WM	torus	toroidal & poloidal: FFT, radial: FDM
WMF	torus	toroidal & poloidal: FFT, radial: FEM
WF2D	torus	toroidal: FFT, poloidal and radial: FEM
WF3D	Cartesian	x, y, z: FEM

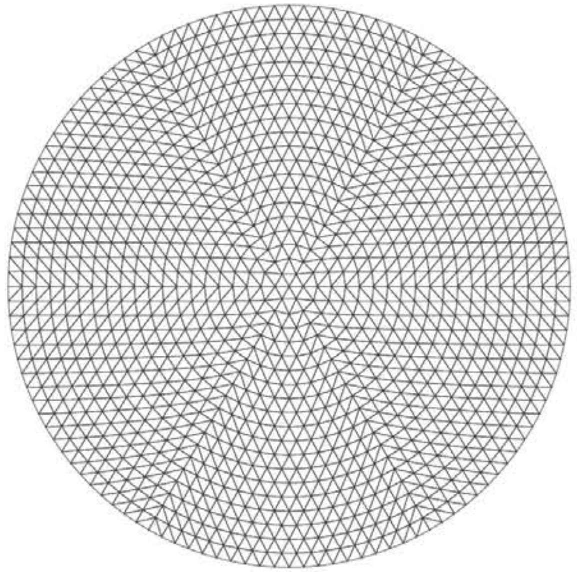
  – Merit of FEM: Flexibility of mesh, sparse matrix, localized analysis

- **Extension of dielectric tensor**
  – Uniform, kinetic, Maxwellian, Fourier expansion
  – Nonuniform, gyro kinetic, Maxwellian, Fourier expansion
  – Nonuniform, kinetic, Maxwellian, Integral form
  – Uniform, kinetic, arbitrary $f(\boldsymbol{v})$, Fourier expansion
  – Nonuniform, gyro kinetic, arbitrary $f(\boldsymbol{v})$, Fourier expansion

- **Coupling with Fokker-Planck analysis of $f(\boldsymbol{v})$**

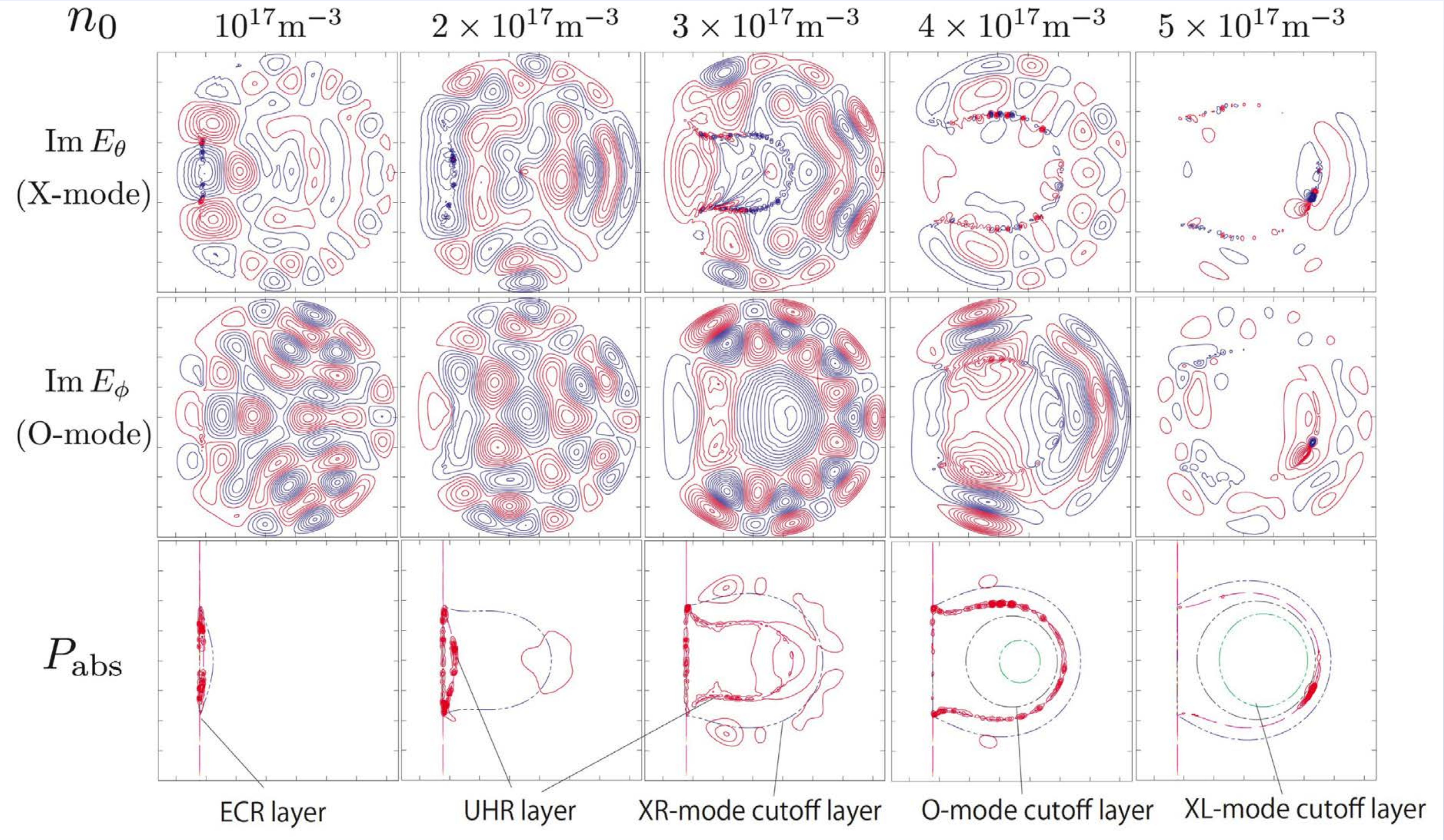
Full wave analysis by FEM: TASK/WF3D/WF2D

- **Wave electric field with complex frequency**: $\tilde{\boldsymbol{E}}(\boldsymbol{r}, t) = \boldsymbol{E}(\boldsymbol{r})\,\mathrm{e}^{-\mathrm{i}\,\omega t}$
- **Maxwell's equation**: $\nabla \times \nabla \times \boldsymbol{E} - \dfrac{\omega^2}{c^2}\overleftrightarrow{\epsilon} \cdot \boldsymbol{E} = \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$
  - $\overleftrightarrow{\epsilon}$: Dielectric tensor
    - Collisional cold plasma model
- **Numerical method**: FEM
  - **3D version**
    - Tetrahedron element
    - Electric field along a edge of a tetrahedron
  - **2D version**: axisymmetric cylindrical
    - Triangular element
    - Scalar (toroidal) and vector (poloidal) hybrid basis function



EC waves in a small-size ST

- R=0.22 m, a=0.16 m, B_0=0.072 T
- f=5 GHz, n_φ=8, ν/ω=0.001 O - X - UHR



Integral Formulation of Wave-Particle Interaction

- **General form of dielectric tensor**

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}, \omega) - \frac{\omega^2}{c^2} \int_V \mathrm{d}\boldsymbol{r}' \overleftrightarrow{\epsilon}(\boldsymbol{r}, \boldsymbol{r}'; \omega) \cdot \boldsymbol{E}(\boldsymbol{r}', \omega) - \mathrm{i}\,\omega\mu_0 \boldsymbol{J}_{\mathrm{ext}}(\boldsymbol{r}, \omega) = \boldsymbol{0}$$

- **Particle orbit**:

$$\boldsymbol{r} = \boldsymbol{r}' + \Delta\boldsymbol{r}(\boldsymbol{v}, \boldsymbol{r}, t - t')$$
$$\boldsymbol{v} = \boldsymbol{v}' + \Delta\boldsymbol{v}(\boldsymbol{v}, \boldsymbol{r}, t - t')$$

- **Perturbed distribution from Vlasov equation**:

$$f(\boldsymbol{r}, \boldsymbol{v}, t) = -\frac{q}{m} \int_{-\infty}^{t} \mathrm{d}t' \left[\boldsymbol{E}(\boldsymbol{r}') + \boldsymbol{v}' \times \boldsymbol{B}(\boldsymbol{r}')\right] \cdot \frac{\partial f_0(\boldsymbol{r}', \boldsymbol{v}')}{\partial \boldsymbol{v}'} \mathrm{e}^{-\mathrm{i}\,\omega t'}$$

- **Induced current**:

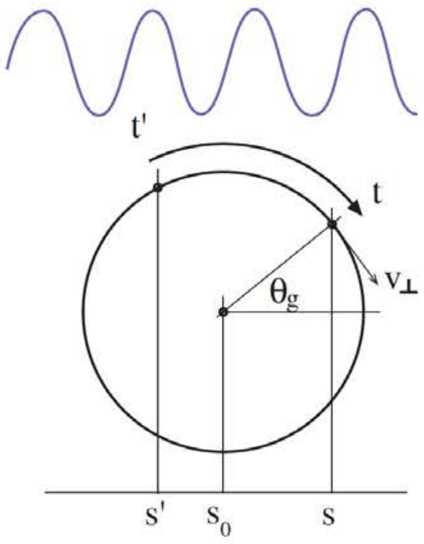
$$\boldsymbol{j}(\boldsymbol{r}) = \int \mathrm{d}\boldsymbol{v}\, q\boldsymbol{v} f(\boldsymbol{r}, \boldsymbol{v}, t)\, \mathrm{e}^{\mathrm{i}\,\omega t} = \int \mathrm{d}\boldsymbol{r}' \overleftrightarrow{\sigma}(\boldsymbol{r} - \boldsymbol{r}', t - t') \cdot E(\boldsymbol{r}')$$

- **The integral form of the conductivity tensor** is defined by

$$\overleftrightarrow{\sigma}(\boldsymbol{r}, \boldsymbol{r}', t-t') = -\frac{q}{m} \int_{-\infty}^{t} \mathrm{d}t' \frac{\partial f_0(\boldsymbol{r}', \boldsymbol{v}')}{\partial \boldsymbol{v}'} \cdot \left[\boldsymbol{v} + \frac{1}{\mathrm{i}\,\omega} \boldsymbol{v} \cdot \boldsymbol{v}' \times \nabla \times\right] \Bigg|_{\substack{\boldsymbol{r}' = \boldsymbol{r} - \Delta\boldsymbol{r}(\boldsymbol{v}, \boldsymbol{r}, t - t') \\ \boldsymbol{v}' = \boldsymbol{v} - \Delta\boldsymbol{v}(\boldsymbol{v}, \boldsymbol{r}, t - t')}}$$

Variable Transformation

- **Transformation of Integral Variables**
  - Transformation from the velocity space variables $(v_\perp, \theta_g)$ to the particle position s' and the guiding center position s_0.



  - Jacobian: $J = \dfrac{\partial(v_\perp, \theta_g)}{\partial(s', s_0)} = -\dfrac{\omega_c^2}{v_\perp \sin \omega_c \tau}$.
  - Express $v_\perp$ and θ_g by s' and s_0 using $\tau = t - t'$, e.g.,

$$v_\perp \sin(\omega_c \tau + \theta_g) = \frac{\omega_c}{v_\perp}\frac{s - s'}{2}\frac{1}{\tan \frac{1}{2}\omega_c \tau} + \frac{\omega_c}{v_\perp}\left(\frac{s + s'}{2} - s_0\right)\tan \frac{1}{2}\omega_c \tau$$

- **Integration over τ**: Fourier expansion with cyclotron motion
- **Integration over $v_{||}$**: Plasma dispersion function
- **Conductivity tensor**: (ℓ : cyclotron harmonics number)

$$\overleftrightarrow{\sigma}(s, s', \chi_0, \zeta_0) = -i n_0 \frac{q^2}{m} \sum_\ell \int \mathrm{d}s_0 \, \overleftrightarrow{H}_\ell(s - s_0, s' - s_0; s_0, \chi_0, \zeta_0)$$

Kernel Functions

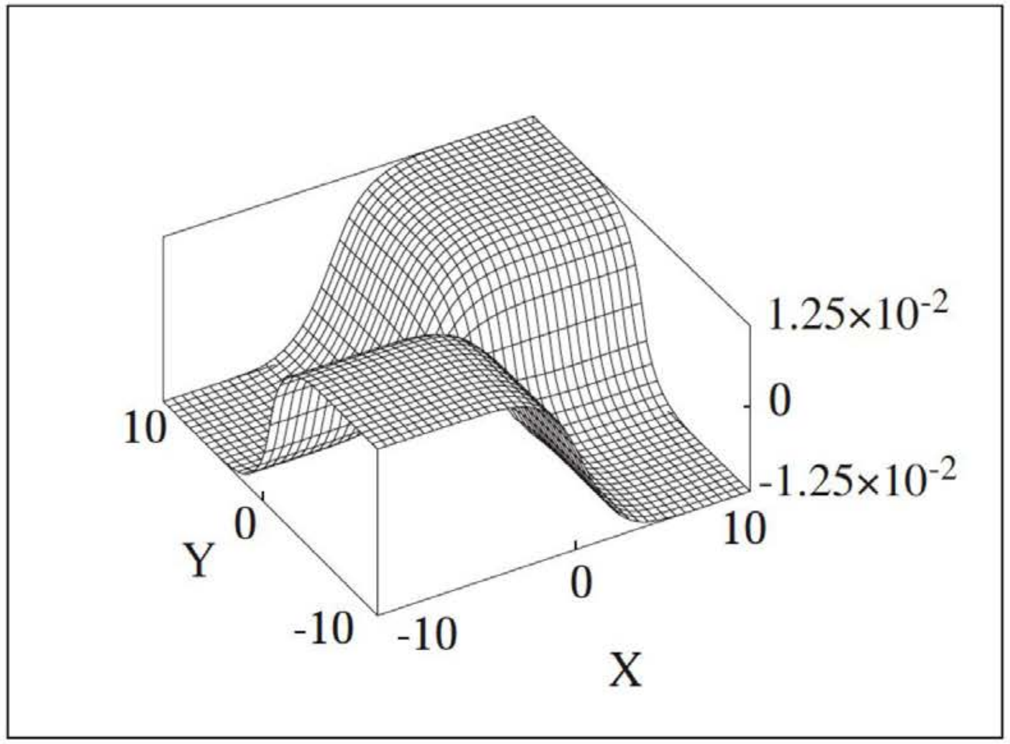
- **Kernel Function H_ℓ and its integral in FEM includes:**

$$F_n^{(i)}(X,Y) \equiv \frac{1}{2\pi^2}\int_0^\pi \mathrm{d}\theta \exp\left[-\frac{X^2}{1+\cos\theta}-\frac{Y^2}{1-\cos\theta}\right] f_n^{(i)}(\theta)$$

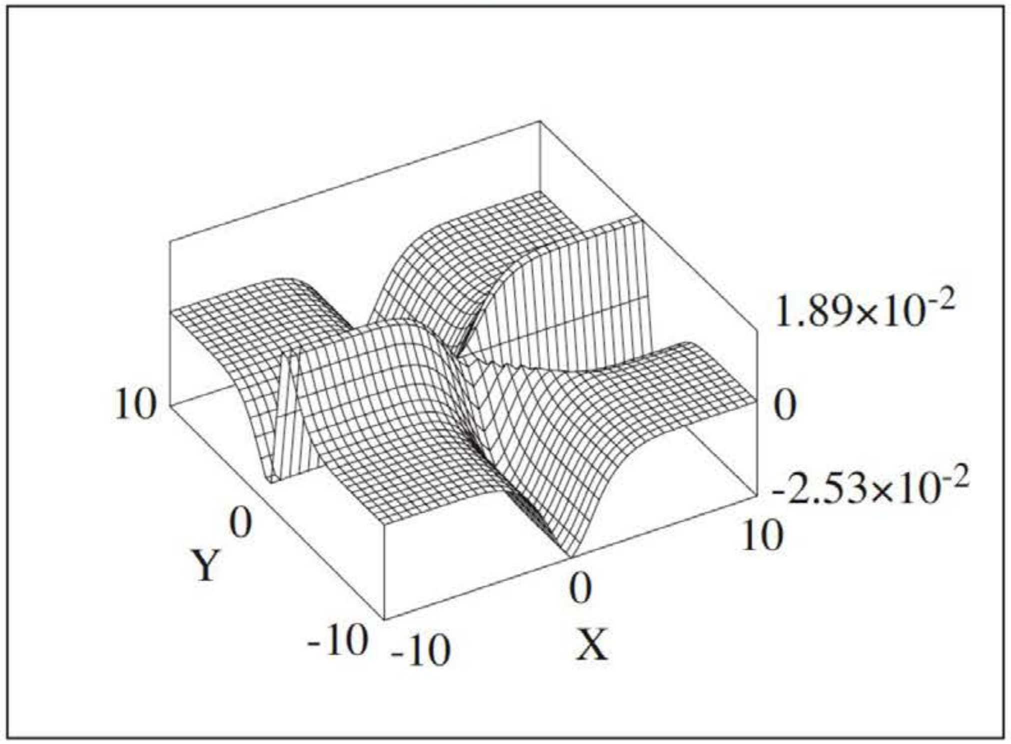
$$\mathcal{F}_n^{(ijk)}(X,Y) \equiv \int_0^Y \mathrm{d}Y' \int_0^{X+Y'} \mathrm{d}X' X'^j Y'^k F_n^{(i)}(X',Y')$$

$$f_n^{(i)}(\theta) = \begin{cases} \dfrac{\cos n\theta}{\sin\theta} & (i=1) \\ \sin n\theta & (i=2) \\ \dfrac{\sin n\theta}{\sin^2\theta} & (i=3) \\ \dfrac{\cos\theta \sin n\theta}{\sin^2\theta} & (i=4) \end{cases}$$

$F_0^{(100)}$



$F_1^{(100)}$

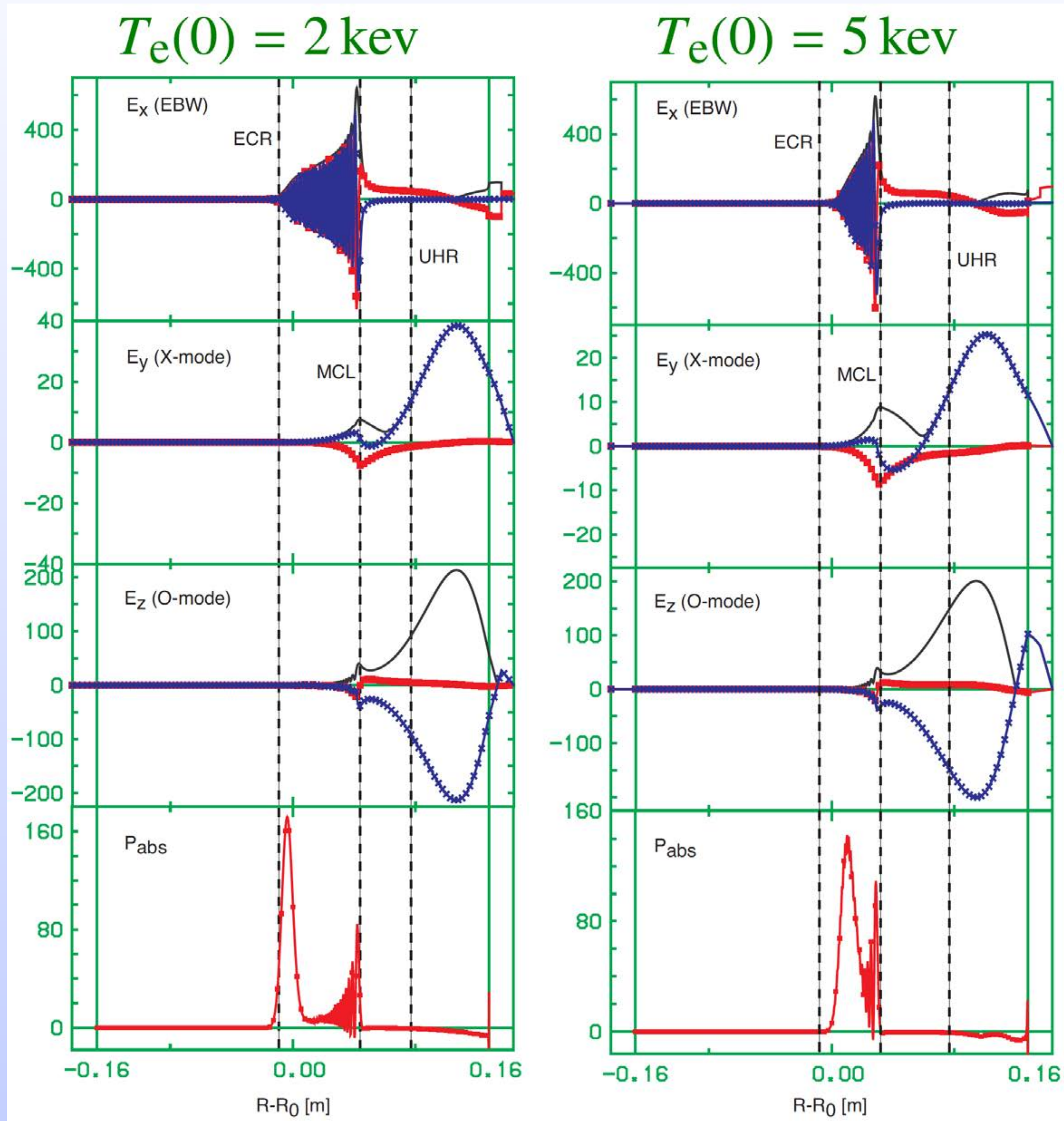


One-Dimensional Analysis

O-X-B excitation

a small-size spherical tokamak

major radius	$R_0 = 0.22\,\mathrm{m}$
minor radius	$a = 0.15\,\mathrm{m}$
central magnetic field	$B_0 = 0.08\,\mathrm{m}$
toroidal mode number	$n_\phi = 24$
central electron density	$3 \times 10^{17}\,\mathrm{m}^{-3}$



Issues in Kinetic Integrated Modeling

- **Modeling of transport process**
  - Turbulent transport coefficients with velocity dependence
  - Finite orbit size effects (Neoclassical transport)
  - Coupling with toroidal electric field (Faraday's law)
  - Keeping charge neutrality (Gauss's Law)
- **Kinetic full wave analysis**
  - Integral form of dielectric tensor including finite gyro radius effects
  - Gyro kinetic dielectric tensor for coupling with drift waves
- **Coupling with other components**
  - Equilibrium including kinetic effects
    - Anisotropic pressure, and flow
  - Modeling of diagnostics
    - Validation by direct comparison

Summary

- Integrated modelling of toroidal plasmas is required for understanding the physics of experimental observations and predicting the performance of future devices.
- For large scale integrated simulation, development and spread of a standard data model is essential. Several efforts to develop infrastructures for integrated modelling are under way.
- We have been developing the integrated modelling suites TASK which includes several levels of transport modelling and full wave analysis of toroidal plasmas.